Question 3

Following Peugh (2010), the following steps were performed in this multilevel modeling (MLM) analysis.

Step 1: Research question

Since we are interested in estimating the effect of two patient level predictors (MI: Myocardial Infarction, and CHF: Congestive Heart Failure) on a patient level outcome (LOS: length of stay) for two independent groups of patients (Clinician, Peer provider) i.e. patients nested within groups, multilevel modeling is justified. Although it is possible to add group dummy as a predictor variable in order to examine the mean difference in LOS between the two groups, observations nested within each group will not be independent of each other thus violating one of the key assumptions of OLS regression.

Step 2: Choice of parameter estimation method

Due to the very small sample size (Level-1, *n* = 44, Level-2, *n* =2) in this problem, the restricted maximum likelihood (REML) method was chosen for parameter estimation. In small samples the alternative estimation method i.e. full information maximum likelihood (FIML) method tends to produce biased variance estimates.

Step 3: Assessment of need for multilevel modeling

Even though the research question suggests that in theory multilevel modeling is justified in this problem, we need to assess the need for such modeling from a practical point of view. In order to determine whether or not multilevel modeling is worthwhile I estimated the intra-class correlation, *ICC* coefficient and the design effect, D_{eff}. Following Raudenbush and Bryk (2002), ICC was calculated by estimating an unconditional hierarchical linear model (HLM) using Level-1 and Level-2 equations shown in (1).

$$\left.\begin{aligned} Level-1: Y_{ij} &= \beta_{0j} + r_{ij} \\ Level-2: \beta_{0j} &= \gamma_{00} + \mu_{0j} \end{aligned}\right\} \qquad (1)$$

The output from HLM model (1) is shown in Figure 3.1.

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Residual		.518056	.113049
Intercept [subject = Group_ID]	Variance	.590457	.868627

a. Dependent Variable: LOS.

Figure 3.1

Based on this unconditional (or random effects ANOVA) model, the variance in LOS at Level-1 was found to be $\sigma^2 = 0.52$ and variance in LOS at Level-2 was found to be $\tau_{00} = 0.59$. These two numbers were used to calculate *ICC* as follows:

$$ICC = \frac{\tau_{00}}{\tau_{00} + \sigma^2} = \frac{0.59}{0.59 + 0.52} = 0.53$$

This suggests that approximately one half (53%) of the variation in LOS is due to differences between groups (Clinician vs Peer provider). This high ICC value (*ICC* > 0.05 [Peugh, 2010]) supports the need for multilevel modeling. Further support for multilevel modeling came from calculation of the design effect (D_{eff} > 2.0 [Peugh, 2010]).

$$D_{eff} = 1 + (n_c - 1)ICC = 1 + \left(\frac{20 + 24}{2} - 1\right)0.53 = 12.13$$

Step 4: Building Level-1 model

In order to control for the effect of MI and CHF, these patient level predictors were added to HLM model shown earlier in (1). The enhanced model is shown in (2).

$$\left.\begin{aligned} Level-1: Y_{ij} &= \beta_{0j} + \beta_{1j} MI + \beta_{2j} CHF + r_{ij} \\ Level-2: \beta_{0j} &= \gamma_{00} + \mu_{0j} \\ \beta_{1j} &= \gamma_{10} \\ \beta_{1j} &= \gamma_{20} \end{aligned}\right\} \qquad (2)$$

The model shown in (2) is known as Level-1 fixed effects model because Level-1 partial slope coefficients are not treated as random variables. It should be noted that the predictors in model (2) were added after group mean centering as recommended by Peugh (2010). Such centering applies regardless of the predictors' scale of measurement. Also, note that MI and CHF are dummy variables that take a value of 1 when respective condition is present in a patient and a value of 0 if the condition is not present. The variance component estimates from model (2) are presented in Figure 3.2. These estimates suggest that inclusion of Level-1 predictors reduced the within-groups variation from the earlier estimate of 0.52 to 0.04. In other words, inclusion of MI and CHF as Level-1 predictors in the HLM model helped explained $\frac{(0.52-0.04)}{0.52}\times 100 = 92.3\%$ of the within-groups variation. This percentage is conceptually comparable to the R^2 statistic from OLS regression predicting LOS from MI and CHF.

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Residual		.042865	.009585
Intercept [subject = Group_ID]	Variance	.612237	.868612

a. Dependent Variable: LOS.

Figure 3.2

The Level-1 fixed effects model presented in (2) is based on the assumption that the effect of Level-1 predictors on LOS does not vary across the Clinician and Peer provider groups. Relaxing this assumption results in a relatively more complex version of the HLM model which is typically referred to as Level-1 random effects model. This random effects model is presented in (3).

$$\left.\begin{aligned} Level-1: Y_{ij} &= \beta_{0j} + \beta_{1j} MI + \beta_{2j} CHF + r_{ij} \\ Level-2: \beta_{0j} &= \gamma_{00} + \mu_{0j} \\ \beta_{1j} &= \gamma_{10} + \mu_{1j} \\ \beta_{1j} &= \gamma_{20} + \mu_{2j} \end{aligned}\right\} \qquad (3)$$

Model (3) differs from model (2) as it treats all Level-1 partial slope coefficients as random variables. Although model (3) is more sophisticated than model (2), computational errors were encountered in its estimation. Peugh (2010) describes various reasons why a computer program may fail to estimate the Level-1 random effects model, and also suggests some strategies to resolve such computational issues. These strategies include (a) raising the number of iterations used in parameter estimation, (b) artificially increasing the variance in dependent variables and at the same time decreasing the variance in independent variables, and (c) simplifying the model by decreasing the number of random effects to be estimated. The first two strategies did not work for the data in this problem. In order to apply the third strategy I estimated two subsets of model (3). These subsets are presented in (4) and (5). Neither of these subsets turned out to be computationally feasible. For this reason I decided to revert back to the Level-1 fixed effects model (model [2]).

$$\left.\begin{aligned} Level-1: Y_{ij} &= \beta_{0j} + \beta_{1j} MI + \beta_{2j} CHF + r_{ij} \\ Level-2: \beta_{0j} &= \gamma_{00} + \mu_{0j} \\ \beta_{1j} &= \gamma_{10} + \mu_{1j} \\ \beta_{1j} &= \gamma_{20} \end{aligned}\right\} \qquad (4)$$

$$\left.\begin{aligned} Level-1: Y_{ij} &= \beta_{0j} + \beta_{1j} MI + \beta_{2j} CHF + r_{ij} \\ Level-2: \beta_{0j} &= \gamma_{00} + \mu_{0j} \\ \beta_{1j} &= \gamma_{10} \\ \beta_{1j} &= \gamma_{20} + \mu_{2j} \end{aligned}\right\} \qquad (5)$$

The parameter estimates from model (2) are presented in Figure 3.3 and suggest that $\gamma_{00} = 4.85$, $p >$ .05; $\gamma_{10} = 1.29$, $p <$ .001; and $\gamma_{20} = 0.90$, $p <$ .001. These results suggest that when patients are nested within provider groups (1) the partial effect of MI on LOS is significant, (2) the partial effect of CHF on LOS is significant, and (3) after controlling for MI and CHF the average LOS for all groups is 4.85 days (this number has a large standard error due to an unusually small number of Level-2 observations [n = 2 groups] as a result of which the p value is larger than 0.05).

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	4.845672	.554167	1.000	8.744	.072	-2.195688	11.887033
MI_centered	1.293758	.070887	40.000	18.251	.000	1.150491	1.437026
CHF_centered	.904086	.072569	40.000	12.458	.000	.757420	1.050753

a. Dependent Variable: LOS.

Figure 3.3

The descriptive statistics for predicted LOS by group are presented in Figure 3.4. These statistics suggest that the Clinician group has a longer LOS (*M* = 5.40 days) compared to the Peer provider group (*M* = 4.29 days). The average of these means is 4.85 which was earlier reported as $\gamma_{00} = 4.85$ in Figure 3.3.

Predicted_LOS

Group	N	Minimum	Maximum	Mean	Std. Deviation
Clinician	20	4.57	5.86	5.3981	.54269
Peer_Provider	24	2.93	5.13	4.2933	.79257

Figure 3.4

For comparison, OLS multiple regression results predicting LOS from MI, CHF, and group membership (1 = Clinician, 0 = Peer provider) are presented in Figure 3.5. These results suggest that the LOS for Clinician group exceeds that of Peer provider group by 0.74 days. The comparable difference in LOS between these groups from the HLM model was 5.40 – 4.29 = 1.11 days.

Step 5: Building Level-2 model

Building a Level-2 model involves adding group level predictors that can help explain the between-groups variation in LOS. Since no such predictors are provided in the source data this step can be skipped.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.975[a]	.951	.948	.207

a. Predictors: (Constant), Group_dummy, CHF, MI

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	33.444	3	11.148	260.077	.000[b]
	Residual	1.715	40	.043		
	Total	35.159	43			

a. Dependent Variable: LOS

b. Predictors: (Constant), Group_dummy, CHF, MI

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	2.929	.081		36.225	.000
	MI	1.294	.071	.686	18.251	.000
	CHF	.904	.073	.438	12.458	.000
	Group_dummy	.738	.068	.411	10.919	.000

a. Dependent Variable: LOS

Figure 3.5

Step 6: Effect size reporting

Estimates of within-groups variation and between-groups variation were presented earlier in Figures 3.1 and 3.2, and can be used to estimate the total amount of explained variance. Since there were no Level-2 predictors in our HLM model, 0% of the between-groups variation in LOS was explained by the model. Level-1 predictors MI and CHF taken together explained 92.3% of the within-groups variation in LOS. Thus, of the total variation in LOS the HLM model as a whole explained approximately 43.4%.

$$\begin{aligned} \%\ \text{of explained variation} &= ICC \times 0\% + (1 - ICC) \times 92.3\% \\ &= 0.53 \times 0\% + (1 - 0.53) \times 92.3\% \\ &= 43\% \end{aligned}$$

If we add all of the unexplained between-groups variation (ICC = 53%) to this estimate of 43%, then the total figure of 96% is conceptually comparable to the R^2 value of 95.1% reported in the OLS regression output in figure 3.5 (note that OLS regression treats between-groups variation as explained variation).

Step 7: Likelihood model ratio testing

Model fit statistics from the unconditional and final HLM models are presented in Figure 3.6. The parameter estimates for the two models are presented in Figure 3.7. Following Peugh 92010), the deviance (reported as -2 Log Likelihood or –2LL) and total number of parameters from these results can be used to calculate the observed χ^2 value of 99.8 as shown below.

$$\begin{aligned}\chi^2(5-3) &= [-2LL_{Model1}] - [-2LL_{Model2}] \\ &= 100.79 - 0.99 \\ &= 99.8\end{aligned}$$

Since the observed χ^2 value of 99.8 is larger than the corresponding critical value of 5.99 at .05 level of significance, we can conclude that the HLM model that includes MI and CHF as predictors fits the data significantly better than the unconditional model that did not include any predictors.

Information Criteria[a]

-2 Restricted Log Likelihood	100.786
Akaike's Information Criterion (AIC)	104.786
Hurvich and Tsai's Criterion (AICC)	105.086
Bozdogan's Criterion (CAIC)	110.308
Schwarz's Bayesian Criterion (BIC)	108.308

The information criteria are displayed in smaller-is-better form.

a. Dependent Variable: LOS.

a. Results from model (1)

Information Criteria[a]

-2 Restricted Log Likelihood	.987
Akaike's Information Criterion (AIC)	4.987
Hurvich and Tsai's Criterion (AICC)	5.303
Bozdogan's Criterion (CAIC)	10.414
Schwarz's Bayesian Criterion (BIC)	8.414

The information criteria are displayed in smaller-is-better form.

a. Dependent Variable: LOS.

b. Results from model (2)

Figure 3.6

Model Dimension[a]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
Random Effects	Intercept[b]	1	Variance Components	1	Group_ID
Residual				1	
Total		2		3	

a. Dependent Variable: LOS.

a. Results from model (1)

Model Dimension[a]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	MI_centered	1		1	
	CHF_centered	1		1	
Random Effects	Intercept[b]	1	Variance Components	1	Group_ID
Residual				1	
Total		4		5	

a. Dependent Variable: LOS.

b. Results from model (2)

Figure 3.7