

# Learning Networks through Regressions

Farrokh Alemi, PhD

# Learning Network Structure

	Outcome Y	Covariate C1	Covariate C2	Exposure X	Mediator Z
Intercept	0.3	0.6	0.85	0.12	0.45
Covariate C1	0.12			0.85	
Covariate C2	0.22			0.35	
Exposure X	0.15				0.82
Mediator, Z	0.4				
McFadden R2	0.18	0.12	0.13	0.23	0.34

# Learning Network Structure

	Outcome Y
Intercept	0.3
Covariate C1	<b>0.12</b>
Covariate C2	<b>0.22</b>
Exposure X	<b>0.15</b>
Mediator, Z	<b>0.4</b>
McFadden R2	0.18

# Learning Network Structure

	Outcome Y	<b>Covariate C1</b>
Intercept	0.3	0.6
Covariate C1	0.12	
Covariate C2	0.22	
Exposure X	0.15	
Mediator, Z	0.4	
McFadden R2	0.18	0.12

# Learning Network Structure

	Outcome Y	Covariate C1	<b>Covariate C2</b>
Intercept	0.3	0.6	0.85
Covariate C1	0.12		
Covariate C2	0.22		
Exposure X	0.15		
Mediator, Z	0.4		
McFadden R2	0.18	0.12	0.13

# Learning Network Structure

	Outcome Y	Covariate C1	Covariate C2	Exposure X
Intercept	0.3	0.6	0.85	0.12
<b>Covariate C1</b>	0.12			<b>0.85</b>
<b>Covariate C2</b>	0.22			<b>0.35</b>
Exposure X	0.15			
Mediator, Z	0.4			
McFadden R2	0.18	0.12	0.13	0.23

# Learning Network Structure

	Outcome Y	Covariate C1	Covariate C2	Exposure X	<b>Mediator Z</b>
Intercept	0.3	0.6	0.85	0.12	0.45
<b>Covariate C1</b>	0.12			0.85	
<b>Covariate C2</b>	0.22			0.35	
<b>Exposure X</b>	0.15				0.82
Mediator, Z	0.4				
McFadden R2	0.18	0.12	0.13	0.23	0.34

# Learning Network Structure

	Outcome Y	Covariate C1	Covariate C2	Exposure X	Mediator Z
Intercept	0.3	0.6	0.85	0.12	0.45
<b>Covariate C1</b>	0.12			0.85	
<b>Covariate C2</b>	0.22			0.35	
<b>Exposure X</b>	0.15				0.82
Mediator, Z	0.4				
McFadden R2	0.18	0.12	0.13	0.23	0.34

0 not shown

# Learning Network Structure

	Outcome Y	Covariate C1	Covariate C2	Exposure X	Mediator Z
Intercept	<b>0.3</b>	<b>0.6</b>	<b>0.85</b>	<b>0.12</b>	<b>0.45</b>
Covariate C1	<b>0.12</b>			<b>0.85</b>	
Covariate C2	<b>0.22</b>			<b>0.35</b>	
Exposure X	<b>0.15</b>				<b>0.82</b>
Mediator, Z	<b>0.4</b>				
McFadden R2	0.18	0.12	0.13	0.23	0.34

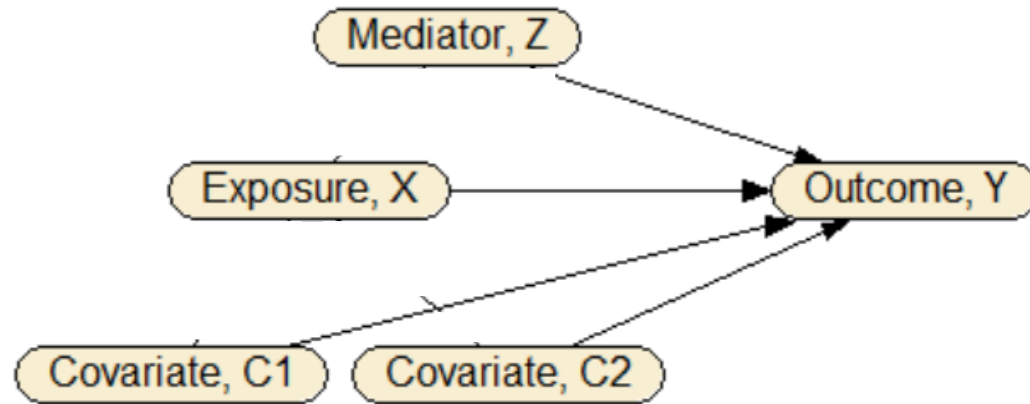
# Learning Network Structure

	Outcome	Covariate C1	Covariate C2	Mediator, Z
Intercept	0.3	0.6	0.85	
Covariate C1	0.1		0.1	
Covariate C2	0.2		0.35	
Exposure X	0.15			0.82
Mediator, Z	0.4			
McFadden R2	0.18	0.12	0.13	0.23

# Learning Network Structure

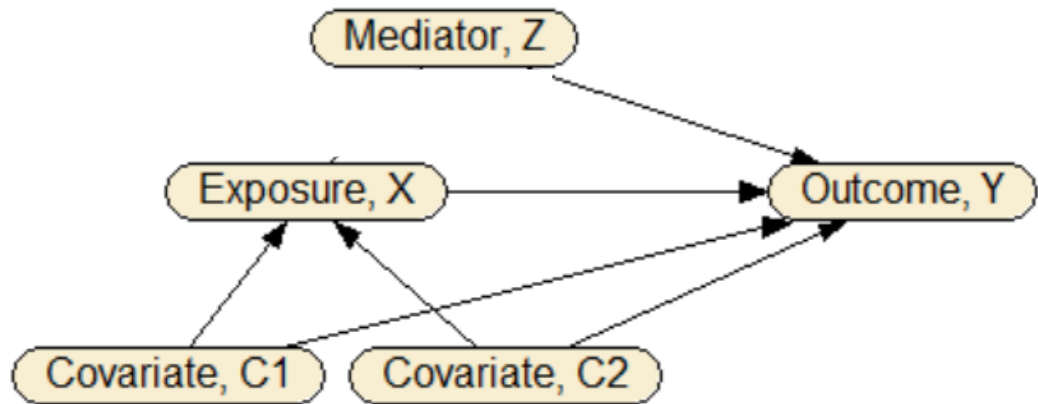
	Outcome Y	Covariate C1	Covariate C2	Exposure X	Mediator Z
Intercept	0.3	0.6	0.85	0.12	0.45
Covariate C1	0.12			0.85	
Covariate C2	0.22			0.35	
Exposure X	0.15				0.82
Mediator, Z	0.4				
<b>McFadden R2</b>	<b>0.18</b>	<b>0.12</b>	<b>0.13</b>	<b>0.23</b>	<b>0.34</b>

# Learning Network Structure



	Outcome Y
Intercept	0.3
Covariate C1	<b>0.12</b>
Covariate C2	<b>0.22</b>
Exposure X	<b>0.15</b>
Mediator, Z	<b>0.4</b>
McFadden R2	0.18

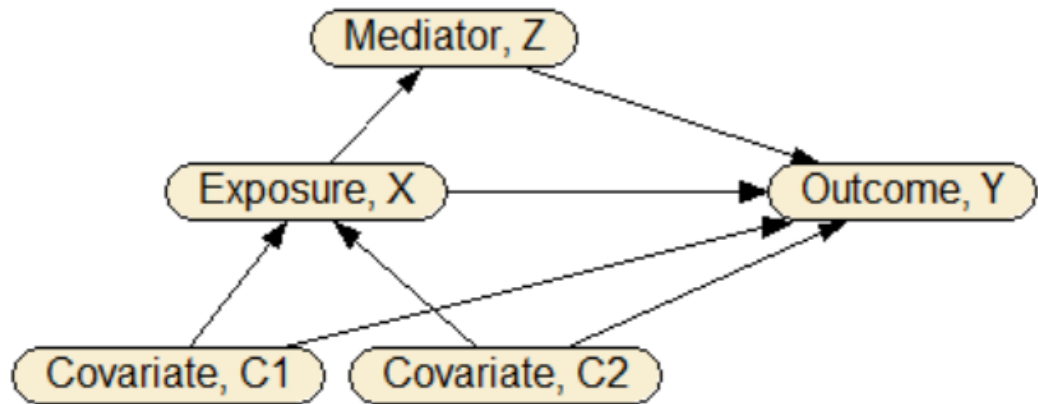
# Learning Network Structure



	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

	Exposure X
Intercept	0.12
Covariate C1	<b>0.85</b>
Covariate C2	<b>0.35</b>
Exposure X	
Mediator, Z	
McFadden R2	0.23

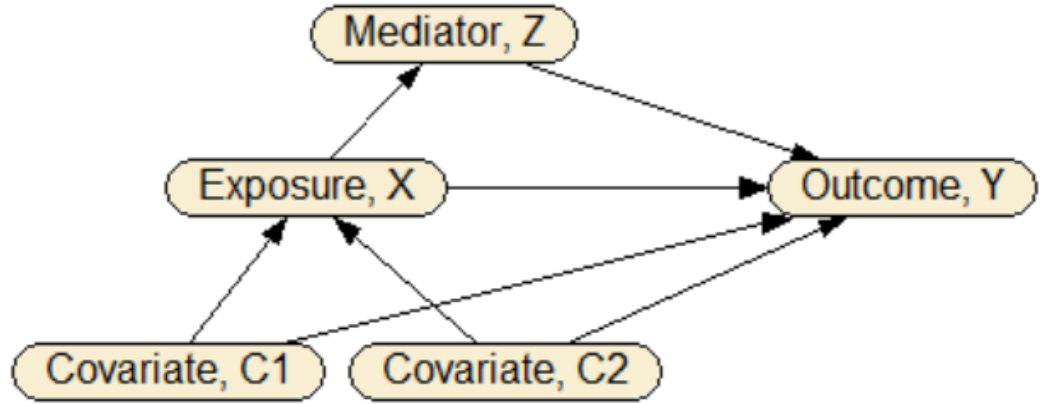
# Learning Network Structure



	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

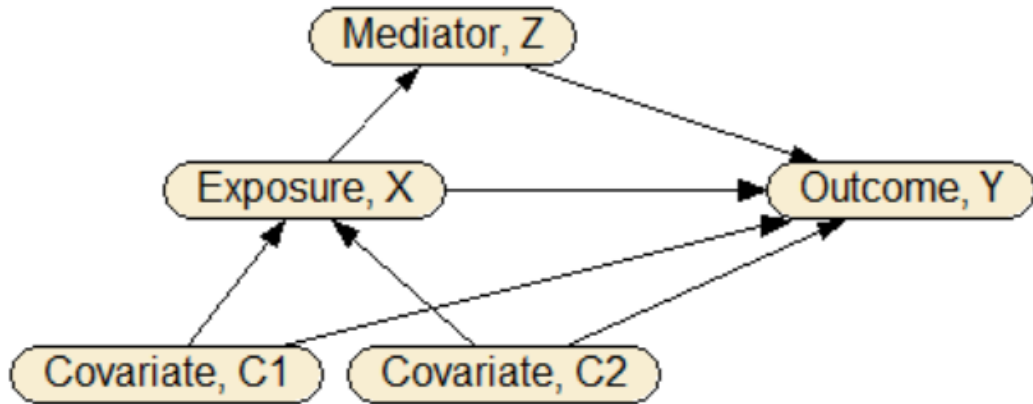
Exposure X	Mediator Z
0.12	0.45
0.85	
0.35	
	<b>0.82</b>
0.23	0.34

# Learning Network Structure



	Outcome Y	Covariate C1	Covariate C2	Exposure X	Mediator Z
Intercept	0.3	0.6	0.85	0.12	0.45
Covariate C1	0.12			0.85	
Covariate C2	0.22			0.35	
Exposure X	0.15				0.82
Mediator, Z	0.4				
McFadden R2	0.18	0.12	0.13	0.23	0.34

# Estimating Distribution of Y



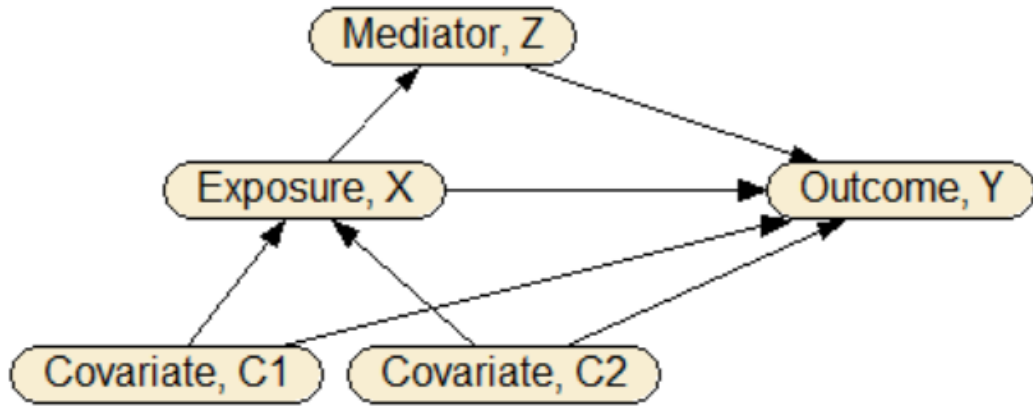
	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18







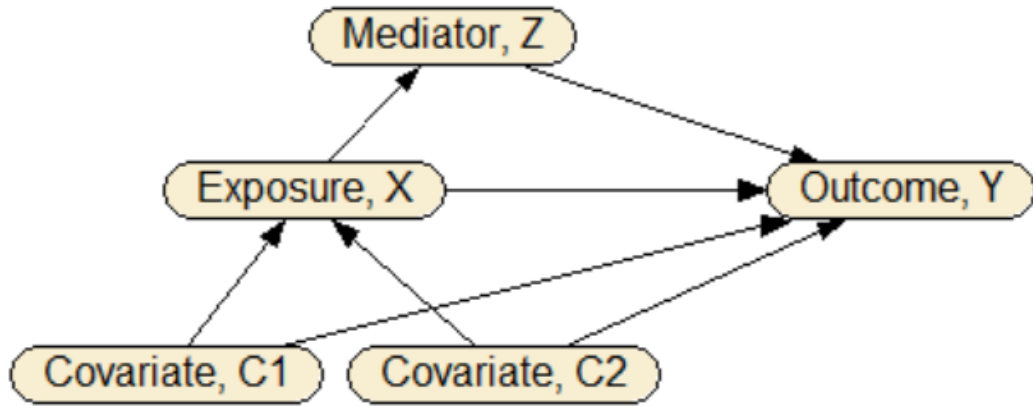
# Estimating Distribution of Y



	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1			
1	1			
1	1			
1	1			
1	0			
1	0			
1	0			
1	0			
0	1			
0	1			
0	1			
0	0			
0	0			
0	0			
0	0			

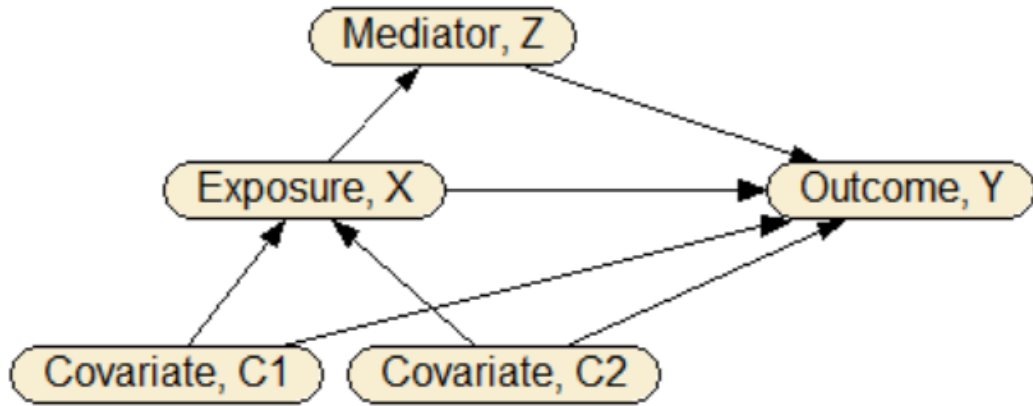
# Estimating Distribution of Y



	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1		
1	1	1		
1	1	0		
1	1	0		
1	0			
1	0			
1	0			
1	0			
0	1			
0	1			
0	1			
0	0			
0	0			
0	0			
0	0			

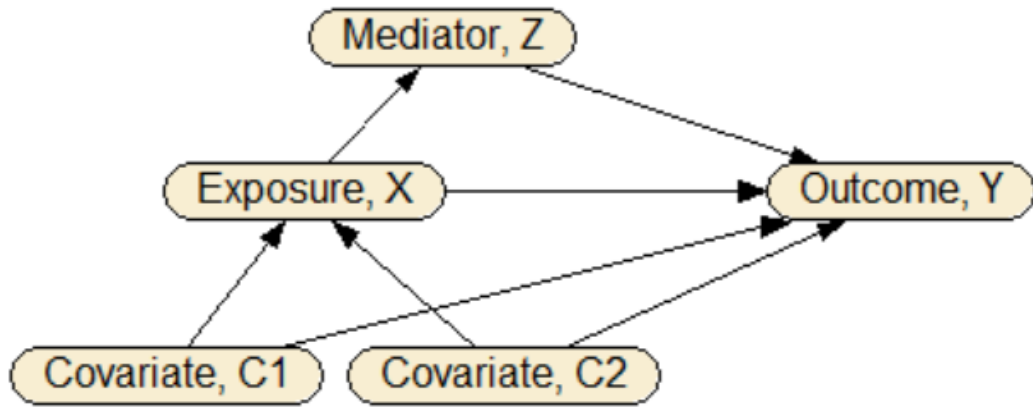
# Estimating Distribution of Y



	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1		
1	1	1		
1	1	0		
1	1	0		
1	0	1		
1	0	1		
1	0	0		
1	0	0		
0	1			
0	1			
0	1			
0	1			
0	0			
0	0			
0	0			
0	0			

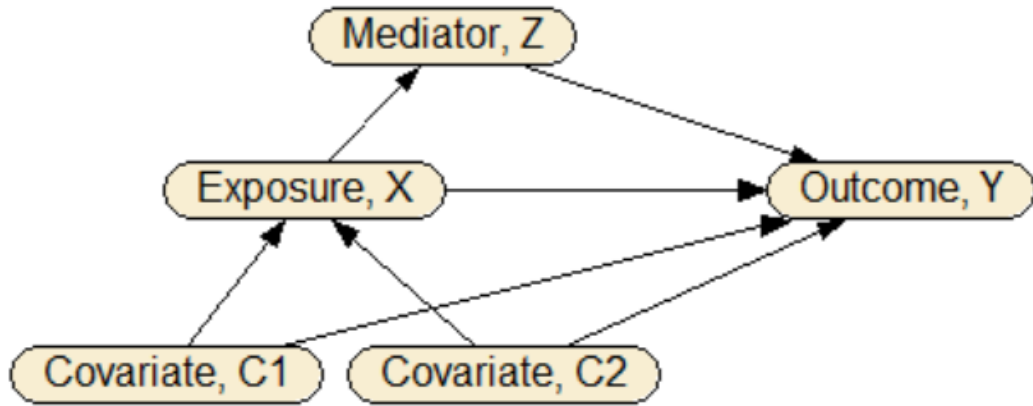
# Estimating Distribution of Y



	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1		
1	1	1		
1	1	0		
1	1	0		
1	0	1		
1	0	1		
1	0	0		
1	0	0		
0	1	1		
0	1	1		
0	1	0		
0	1	0		
0	0	1		
0	0	1		
0	0	0		
0	0	0		

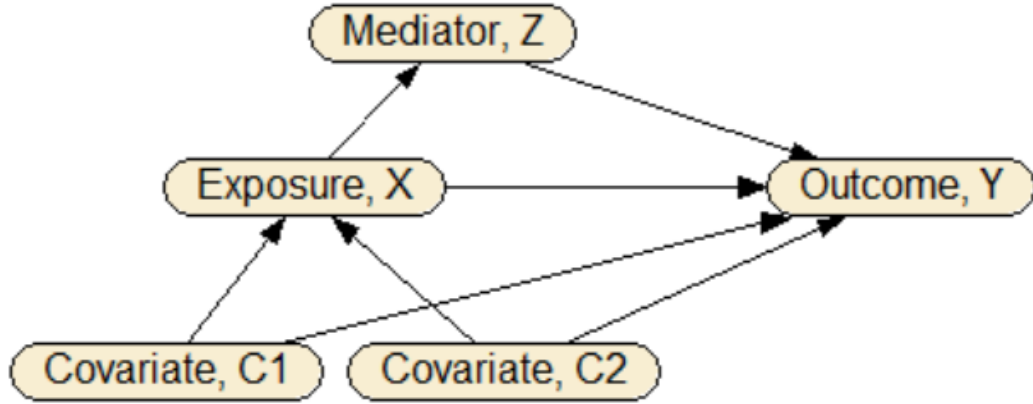
# Estimating Distribution of Y



	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1	1	
1	1	1	0	
1	1	0	1	
1	1	0	0	
1	0	1	1	
1	0	1	0	
1	0	0	1	
1	0	0	0	
0	1	1	1	
0	1	1	0	
0	1	0	1	
0	1	0	0	
0	0	1	1	
0	0	1	0	
0	0	0	1	
0	0	0	0	

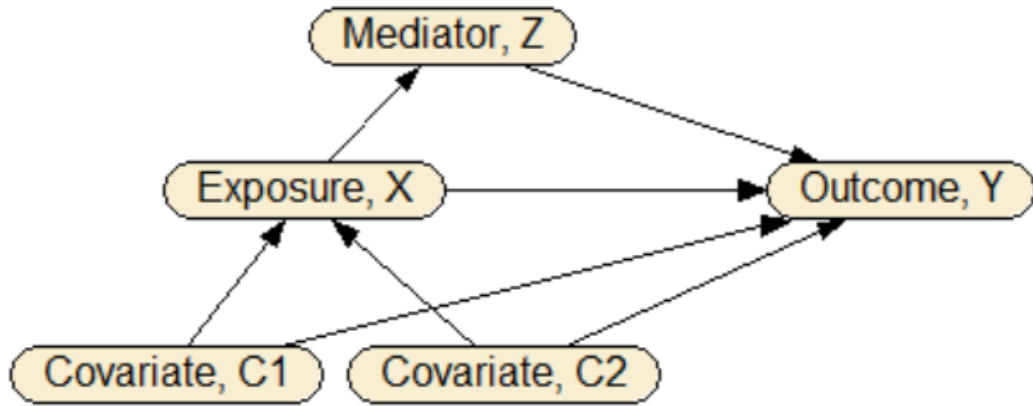
# Estimating Distribution of Y



	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1	1	
1	1	1	0	
1	1	0	1	
1	1	0	0	
1	0	1	1	
1	0	1	0	
1	0	0	1	
1	0	0	0	
0	1	1	1	
0	1	1	0	
0	1	0	1	
0	1	0	0	
0	0	1	1	
0	0	1	0	
0	0	0	1	
0	0	0	0	

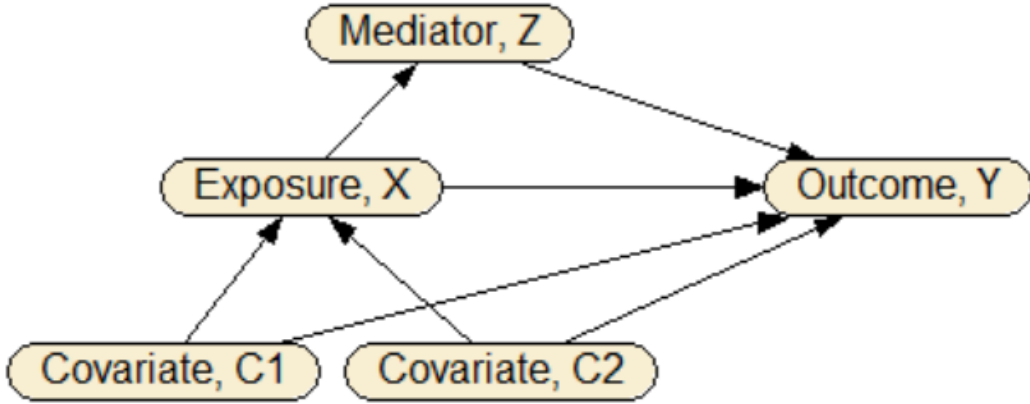
# Estimating Distribution of Y



	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1	1	
1	1	1	0	
1	1	0	1	
1	1	0	0	
1	0	1	1	
1	0	1	0	
1	0	0	1	
1	0	0	0	
0	1	1	1	
0	1	1	0	
0	1	0	1	
0	1	0	0	
0	0	1	1	
0	0	1	0	
0	0	0	1	
0	0	0	0	

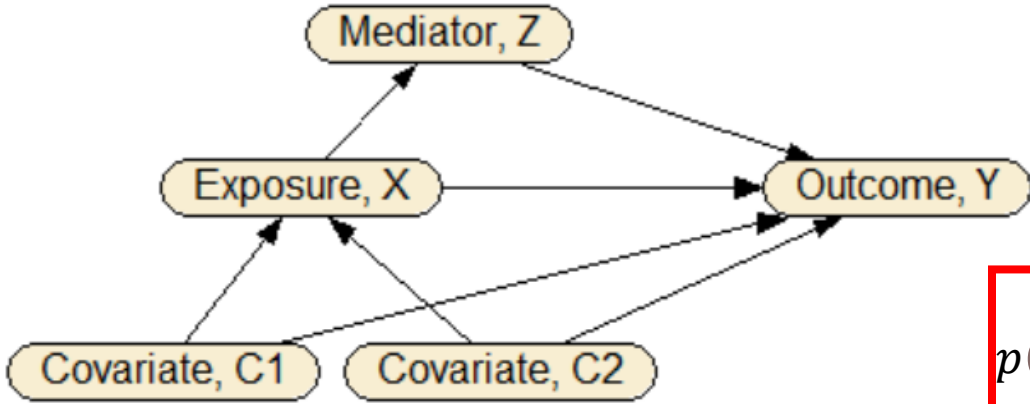
# Estimating Distribution of Y



	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1	1	
1	1	1	0	
1	1	0	1	
1	1	0	0	
1	0	1	1	
1	0	1	0	
1	0	0	1	
1	0	0	0	
0	1	1	1	
0	1	1	0	
0	1	0	1	
0	1	0	0	
0	0	1	1	
0	0	1	0	
0	0	0	1	
0	0	0	0	

# Estimating Distribution of Y



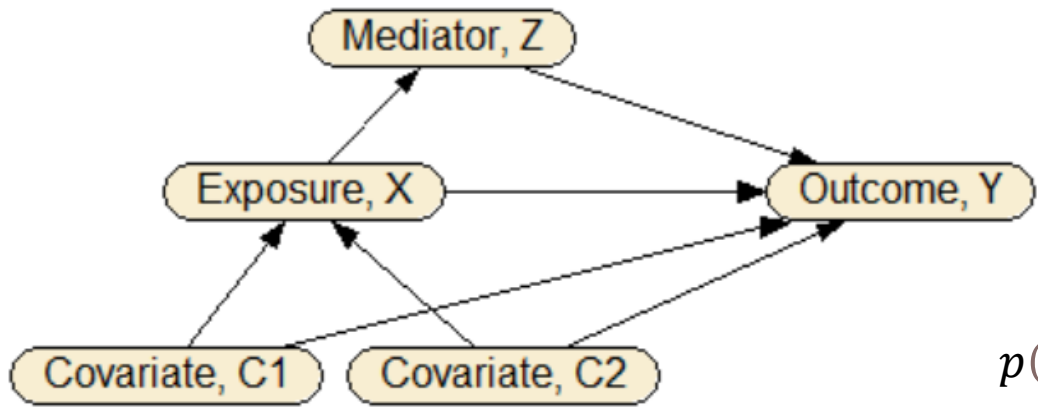
Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1	1	?
1	1	1	0	
1	1	0	1	
1	1	0	0	

$$p(Y = 1) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 C_1 + \beta_2 C_2 + \beta_3 X + \beta_4 Z)}}$$

	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

1	0	0	0	
0	1	1	1	
0	1	1	0	
0	1	0	1	
0	1	0	0	
0	0	1	1	
0	0	1	0	
0	0	0	1	
0	0	0	0	

# Estimating Distribution of Y



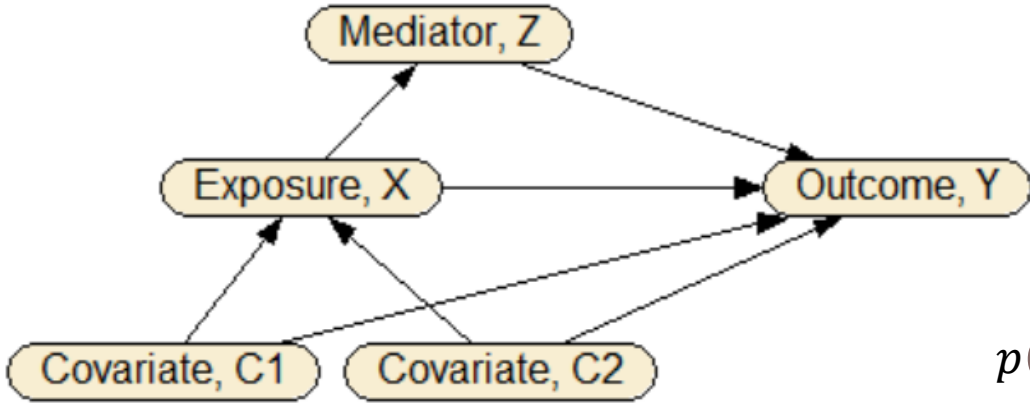
	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1	1	?
1	1	1	0	
1	1	0	1	
1	1	0	0	

$$p(Y = 1) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 C_1 + \beta_2 C_2 + \beta_3 X + \beta_4 Z)}}$$

1	0	0	0	
0	1	1	1	
0	1	1	0	
0	1	0	1	
0	1	0	0	
0	0	1	1	
0	0	1	0	
0	0	0	1	
0	0	0	0	

# Estimating Distribution of Y



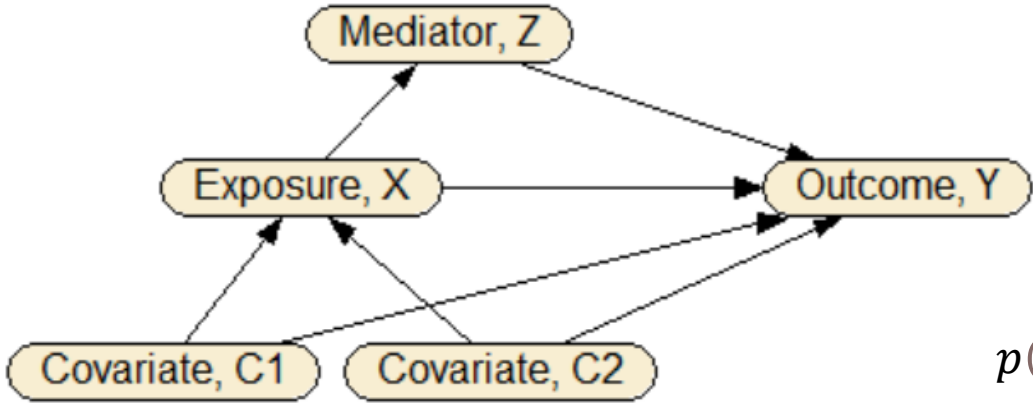
	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1	1	?
1	1	1	0	
1	1	0	1	
1	1	0	0	

$$p(Y = 1) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 C_1 + \beta_2 C_2 + \beta_3 X + \beta_4 Z)}}$$

1	0	0	0	
0	1	1	1	
0	1	1	0	
0	1	0	1	
0	1	0	0	
0	0	1	1	
0	0	1	0	
0	0	0	1	
0	0	0	0	

# Estimating Distribution of Y



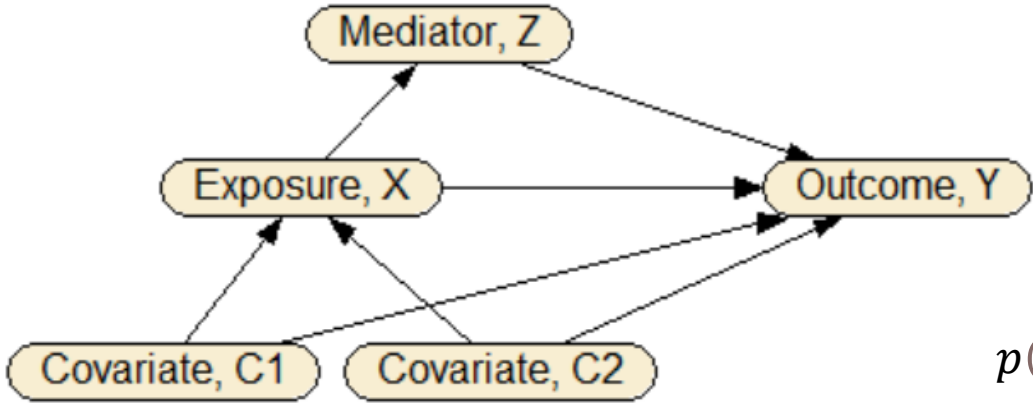
	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1	1	?
1	1	1	0	
1	1	0	1	
1	1	0	0	

$$p(Y = 1) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 C_1 + \beta_2 C_2 + \beta_3 X + \beta_4 Z)}}$$

1	0	0	0	
0	1	1	1	
0	1	1	0	
0	1	0	1	
0	1	0	0	
0	0	1	1	
0	0	1	0	
0	0	0	1	
0	0	0	0	

# Estimating Distribution of Y



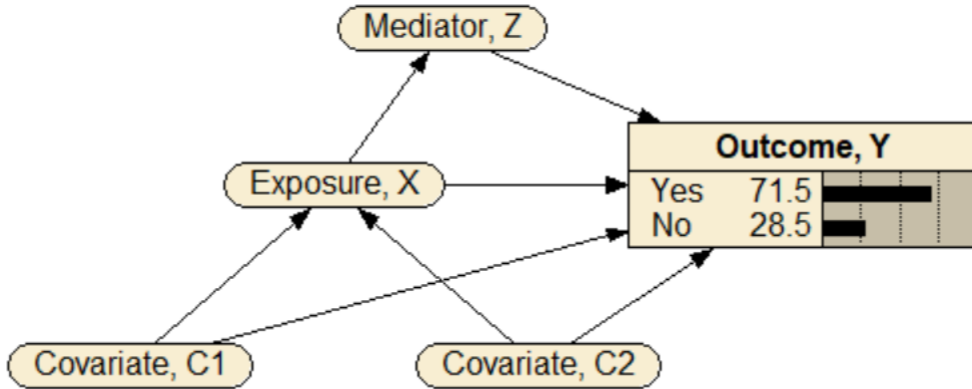
	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1	1	0.77
1	1	1	0	
1	1	0	1	
1	1	0	0	

$$p(Y = 1) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 C_1 + \beta_2 C_2 + \beta_3 X + \beta_4 Z)}}$$

1	0	0	0	
0	1	1	1	
0	1	1	0	
0	1	0	1	
0	1	0	0	
0	0	1	1	
0	0	1	0	
0	0	0	1	
0	0	0	0	

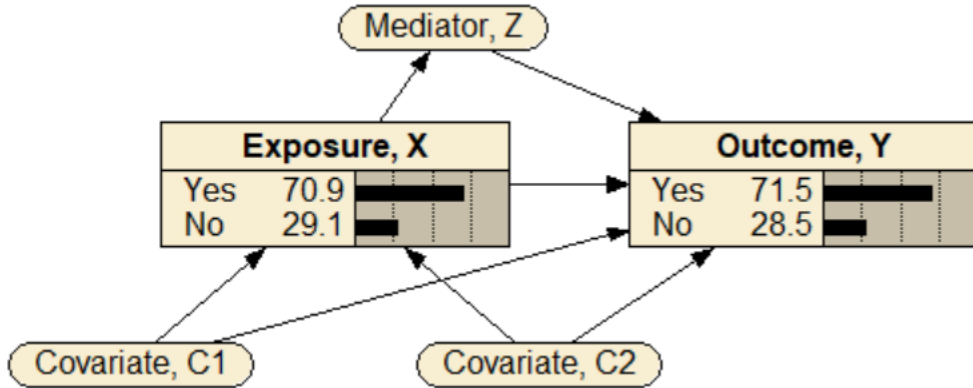
# Estimating Distribution of Y



	Outcome Y
Intercept	0.3
Covariate C1	0.12
Covariate C2	0.22
Exposure X	0.15
Mediator, Z	0.4
McFadden R2	0.18

Covariate C1	Covariate C2	Exposure X	Mediator Z	Outcome Y=1
1	1	1	1	77%
1	1	1	0	69%
1	1	0	1	74%
1	1	0	0	65%
1	0	1	1	73%
1	0	1	0	64%
1	0	0	1	69%
1	0	0	0	60%
0	1	1	1	74%
0	1	1	0	66%
0	1	0	1	72%
0	1	0	0	63%
0	0	1	1	70%
0	0	1	0	61%
0	0	0	1	67%
0	0	0	0	57%

# Estimating Distribution of X

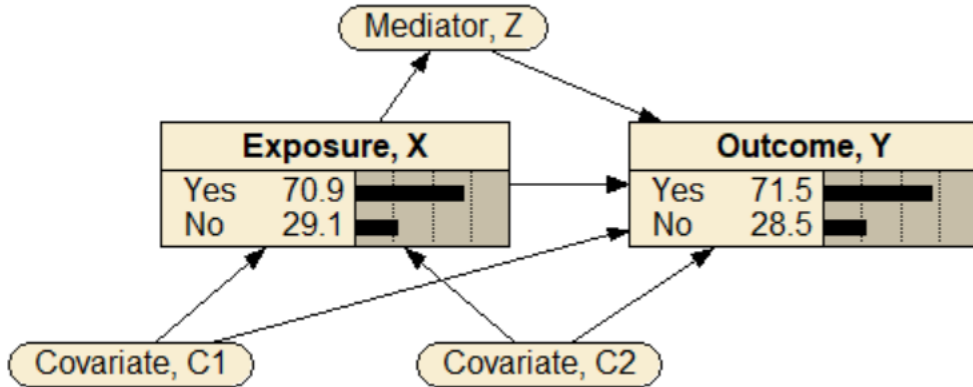


Covariate C1	Covariate C2	Exposure X
1	1	79%
1	0	
0	1	
0	0	

	Exposure X
Intercept	0.12
Covariate C1	0.85
Covariate C2	0.35
Exposure X	
Mediator, Z	
McFadden R2	0.23

$$p(X = 1 | C1 = 1, C2 = 1) = \frac{1}{1 + e^{-(0.12 + 0.85 + 0.35)}} = 79\%$$

# Estimating Distribution of X



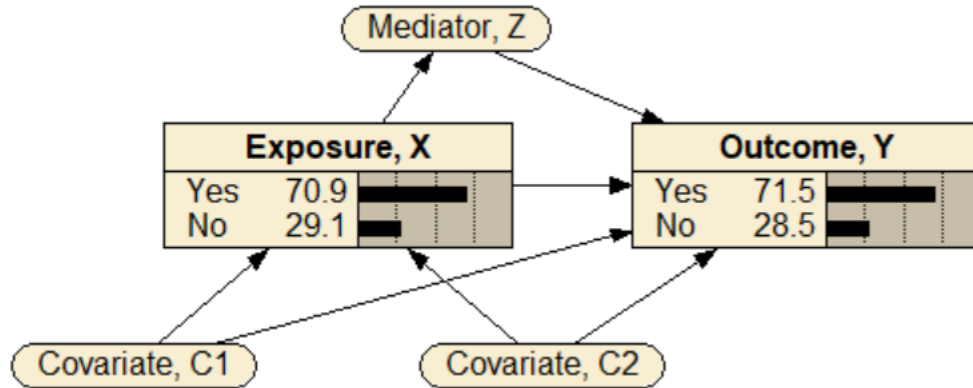
Covariate C1	Covariate C2	Exposure X
1	1	79%
1	0	73%
0	1	
0	0	

	Exposure X
Intercept	0.12
Covariate C1	0.85
Covariate C2	0.35
Exposure X	
Mediator, Z	
McFadden R2	0.23

$$p(X = 1|C1 = 1, C2 = 1) = \frac{1}{1 + e^{-(0.12+0.85+0.35)}} = 79\%$$

$$p(X = 1|C1 = 1, C2 = 0) = \frac{1}{1 + e^{-(0.12+0.85)}} = 73\%$$

# Estimating Distribution of X



Covariate C1	Covariate C2	Exposure X
1	1	79%
1	0	73%
0	1	62%
0	0	53%

	Exposure X
Intercept	0.12
Covariate C1	0.85
Covariate C2	0.35
Exposure X	
Mediator, Z	
McFadden R2	0.23

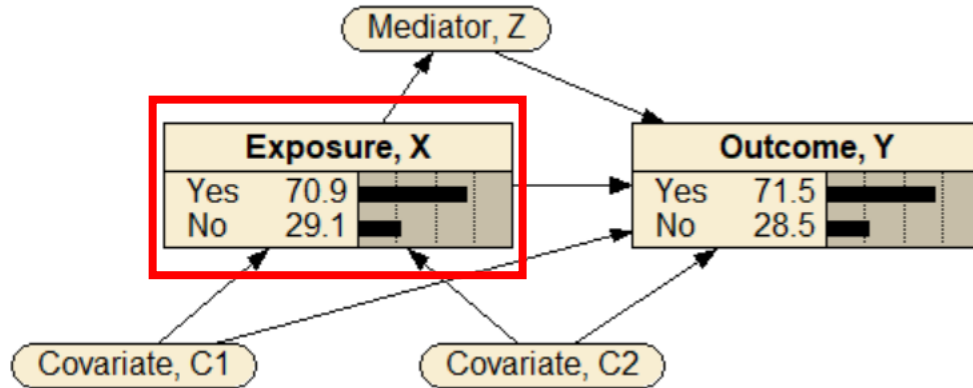
$$p(X = 1|C1 = 1, C2 = 1) = \frac{1}{1 + e^{-(0.12+0.85+0.35)}} = 79\%$$

$$p(X = 1|C1 = 1, C2 = 0) = \frac{1}{1 + e^{-(0.12+0.85)}} = 73\%$$

$$p(X = 1|C1 = 0, C2 = 1) = \frac{1}{1 + e^{-(0.12+0.35)}} = 62\%$$

$$p(X = 1|C1 = 0, C2 = 0) = \frac{1}{1 + e^{-(0.12)}} = 53\%$$

# Estimating Distribution of X



Covariate C1	Covariate C2	Exposure X
1	1	79%
1	0	73%
0	1	62%
0	0	53%

	Exposure X
Intercept	0.12
Covariate C1	0.85
Covariate C2	0.35
Exposure X	
Mediator, Z	
McFadden R2	0.23

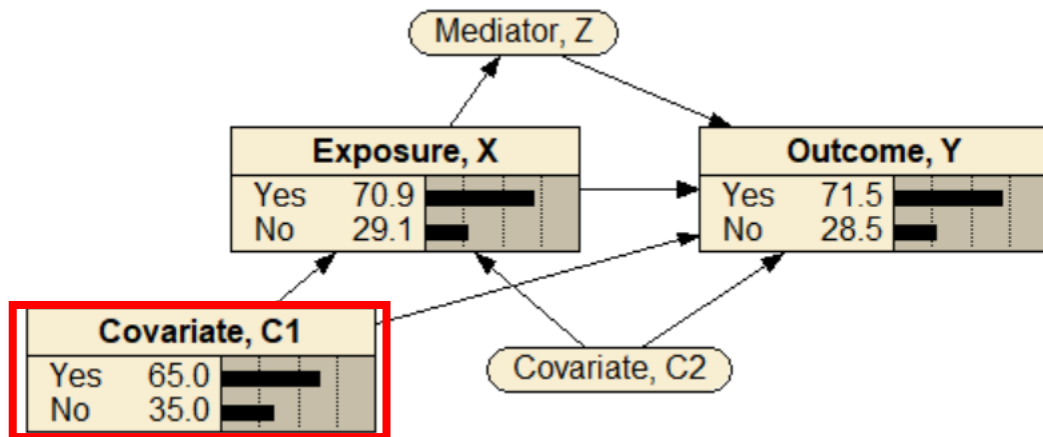
$$p(X = 1|C1 = 1, C2 = 1) = \frac{1}{1 + e^{-(0.12+0.85+0.35)}} = 79\%$$

$$p(X = 1|C1 = 1, C2 = 0) = \frac{1}{1 + e^{-(0.12+0.85)}} = 73\%$$

$$p(X = 1|C1 = 0, C2 = 1) = \frac{1}{1 + e^{-(0.12+0.35)}} = 62\%$$

$$p(X = 1|C1 = 0, C2 = 0) = \frac{1}{1 + e^{-(0.12)}} = 53\%$$

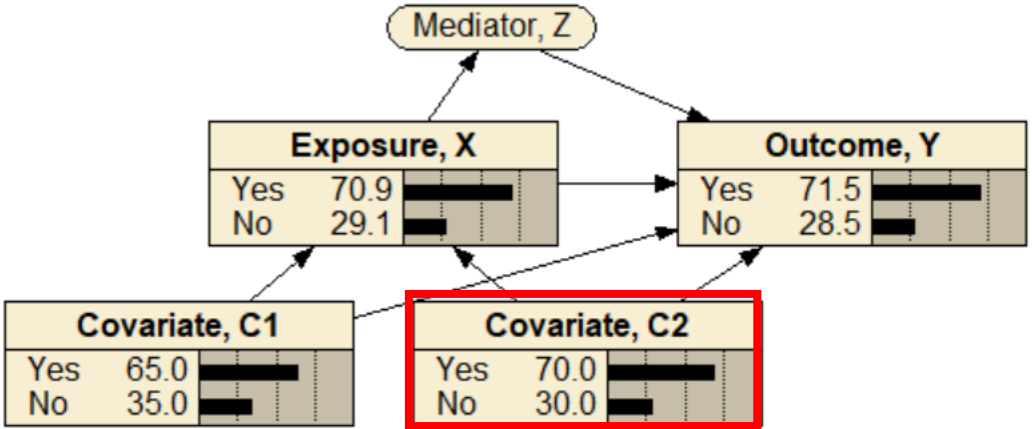
# Estimating C1 Distribution



$$p(C1 = 1) = \frac{1}{1 + e^{-(0.6)}} = 65\%$$

	Covariate C1
Intercept	0.6
Covariate C1	
Covariate C2	
Exposure X	
Mediator, Z	
McFadden R2	0.12

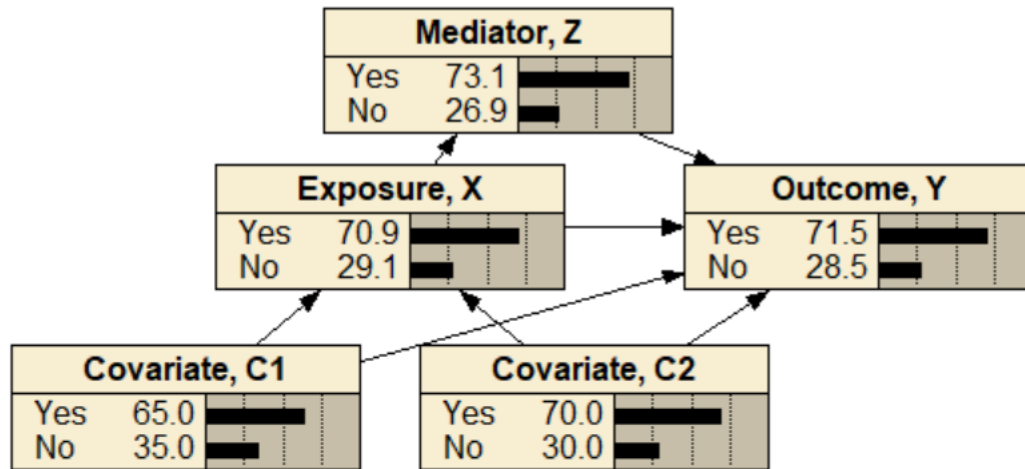
# Estimating C2 Distribution



$$p(C2 = 1) = \frac{1}{1 + e^{-(0.85)}} = 70\%$$

	Covariate C2
Intercept	0.85
Covariate C1	
Covariate C2	
Exposure X	
Mediator, Z	
McFadden R2	0.13

# Estimating Z Distribution

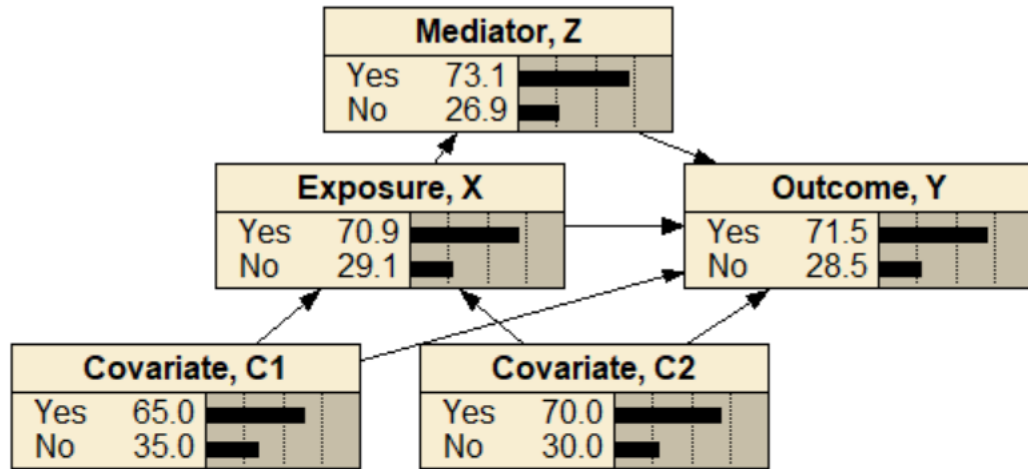


Exposure X	Mediator Z
Yes	78%
No	

	Mediator Z
Intercept	0.45
Covariate C1	
Covariate C2	
Exposure X	0.82
Mediator, Z	
McFadden R2	0.34

$$p(Z = 1|X = 1) = \frac{1}{1 + e^{(-0.45 - 0.82)}} = 78\%$$

# Estimating Z Distribution



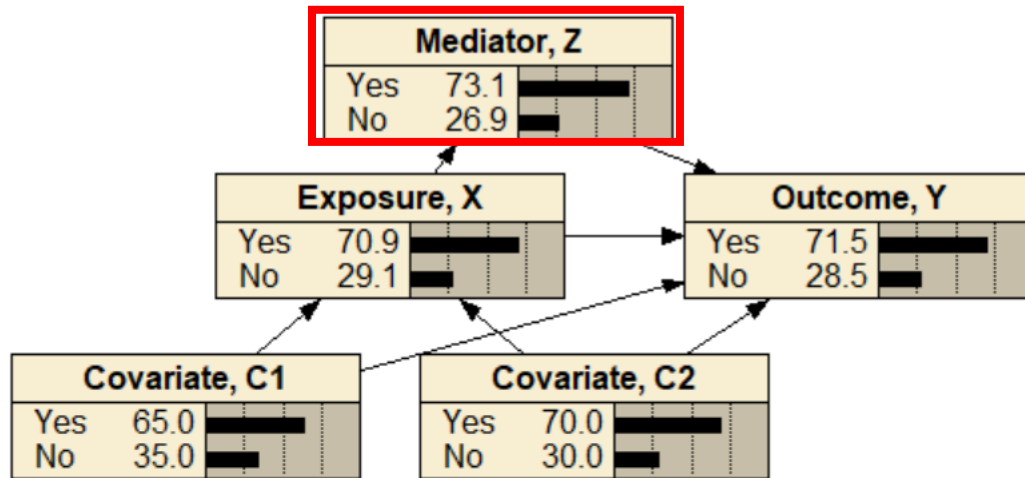
Exposure X	Mediator Z
Yes	78%
No	61%

	Mediator Z
Intercept	0.45
Covariate C1	
Covariate C2	
Exposure X	0.82
Mediator, Z	
McFadden R2	0.34

$$p(Z = 1|X = 1) = \frac{1}{1 + e^{(-0.45 - 0.82)}} = 78\%$$

$$p(Z = 1|X = 0) = \frac{1}{1 + e^{(-0.45)}} = 61\%$$

# Estimating Z Distribution



Exposure X	Mediator Z
Yes	78%
No	61%

	Mediator Z
Intercept	0.45
Covariate C1	
Covariate C2	
Exposure X	0.82
Mediator, Z	
McFadden R2	0.34

$$p(Z = 1|X = 1) = \frac{1}{1 + e^{(-0.45 - 0.82)}} = 78\%$$

$$p(Z = 1|X = 0) = \frac{1}{1 + e^{(-0.45)}} = 61\%$$

# Learned Network

