CHAPTER

5

DECISION TREES

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This chapter introduces decision trees, which are tools for choosing among alternatives. Tools for measuring a decision maker's value and uncertainty were introduced in chapters 2 through 4. Those tools are useful for many problems, but their usefulness is limited when a series of intervening events is likely. When a sequence of events must be analyzed, decision trees provide a means to consider both value and uncertainty.

The first part of this chapter defines decision trees, shows how they are constructed, and describes how they can be analyzed using mathematical expectations. The middle part of the chapter introduces the concept of "folding back," which is useful for analyzing decision trees. The last part of the chapter extends the discussion from a single consequence (i.e., healthcare costs to an employer) to an analysis of multiple consequences. Simple decisions involve one consequence of interest; in the simplest decisions, that is cost. If a decision maker's attitudes toward risk affect the decision, then costs must be transferred to a utility scale. If the decision involves multiple consequences, then the analyst needs to develop an MAV model to transfer the consequences to one scale. These extensions of the method are discussed in the last part of the chapter. In addition, the chapter ends with a discussion of the importance of analyzing the sensitivity of conclusions to input, a topic that will repeatedly be returned to in this and other chapters (e.g., Chapter 8).

The Benefit Manager's Dilemma

This chapter focuses on a dilemma faced by the benefits manager of a bank with 992 employees, all of them covered by an indemnity health insurance program. The benefit manager wants to analyze if a preferred provider arrangement will save the bank healthcare funds. Currently, employees can seek care from any physician and, after satisfying an annual deductible, must pay only a copayment, with the employer paying the remainder. A preferred provider organization (PPO) has approached the benefits manager and has

This book has a companion web site that features narrated presentations, animated examples, PowerPoint slides, online tools, web links, additional readings, and examples of students' work. To access this chapter's learning tools, go to ache.org/DecisionAnalysis and select Chapter 5.

offered to discount services to those employees who use its clinic and hospital. As an inducement, the PPO wants the bank to increase the deductible and/or copayment of employees who use other providers. Employees would still be free to seek care from any provider, but it would cost them more.

The logic of the arrangement is simple: The PPO can offer a discount because it expects a high volume of sales. Nevertheless, the benefits manager wonders what would happen if employees started using the PPO. In particular, an increase in the rate of referrals and clinic visits could easily eat away the savings on the price per visit. A change of physicians could also alter the employees' place of and rate of hospitalization, which would likewise threaten the potential savings.

Before proceeding, some terminology should be clarified. *Discount* refers to the proposed charges at the PPO compared to what the employer would pay under its existing arrangement with the current provider. *Deductible* is a minimum sum that must be exceeded before the health plan picks up the bill. *Copayment* is the portion of the bill the employee must pay after the deductible is exceeded.

Describing the Problem

A decision tree is a visual tool that shows the sequence of events tells the central line of the story. If the analysis ignores these intervening events, then the sequence and the related story are lost. It would be like reading the beginning and ending of a novel; it may be effective at getting the message across but not at communicating the story.

Imagine a tree with a root, a trunk, and many branches. Lay it on its side, and you have an image of a decision tree. The word "tree" has a special meaning in graph theory. Branches of the decision tree do not lead to the root, trunk, or other branches. Thus, a decision tree is not circular; you cannot begin at one place, travel along the tree, and return to the same place. Because a decision tree shows the temporal sequence—events to the left happen before events to the right—it is described as starting with the leftmost node.

The first part of the decision tree is the *root*. The root of the decision tree, placed to the left and shown as a small square, represents a decision. There are at least two lines emanating from this decision node. Each line corresponds to one option. In Figure 5.1, two lines represent the options of signing a contract with the PPO or continuing with the current plan.

The second component of a decision tree consists of the *chance nodes*. These nodes show the events over which the decision maker has no direct control. From a chance node, several lines are drawn, each showing a different possible event. Suppose, for example, that joining the PPO will change the utilization of hospital and outpatient care. Figure 5.2 portrays these events.

Note that the chance nodes are identified by circles. The distinction between circles and boxes indicates whether the decision maker has control over the events that follow a node. Figure 5.2 suggests that, for people who join the PPO, there is an unspecified probability of hospitalization, outpatient care, or no utilization. These probabilities are shown as P_1, P_2, . . . , P_6; it is the practice to place probabilities above the lines leading to the events they are concerned with.

The third element in a decision tree consists of the *consequences*. While the middle of the decision tree shows events following the decision, the right side (at the end of the branches) shows the consequences of these events. Suppose, for the sake of simplicity, that the benefits manager is only interested in costs to the employer, which exclude copayments and deductibles paid by the employee. Hospital and clinic charges are labeled C_1, C_2, . . . , C_6 and are shown in Figure 5.3.

Figure 5.3 represents the three major elements of a decision tree: decisions, possible events, and consequences (in this case, costs). Also, it is important to keep in mind that a decision tree contains a temporal sequence—events at the left precede events on the right.

Solicitation Process

A decision tree, once analyzed and reported, indicates a preferred option and the rationale for choosing it. Such a report communicates the nature of the decision to other members of the organization. The decision tree and the final report on the preferred option are important organizational documents that can influence people, for better or worse, long after the original decision makers have left.

While the analysis and the final report are important by themselves, the process of gathering data and modifying the decision tree are equally important—perhaps more so. The process helps in several ways:

FIGURE 5.1
An Example of a Decision Node

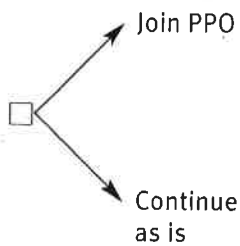
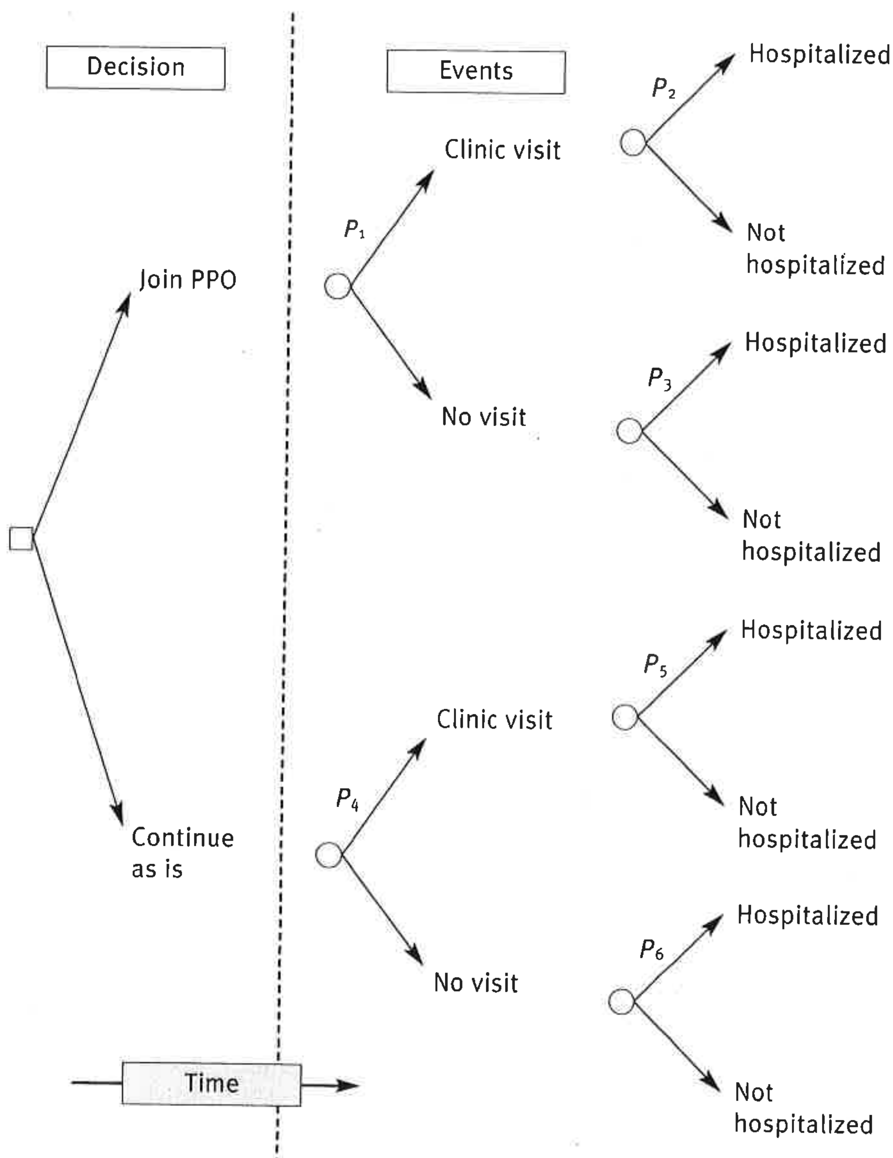


FIGURE 5.2
Possible Events Are Placed to the Right of the Decision Node



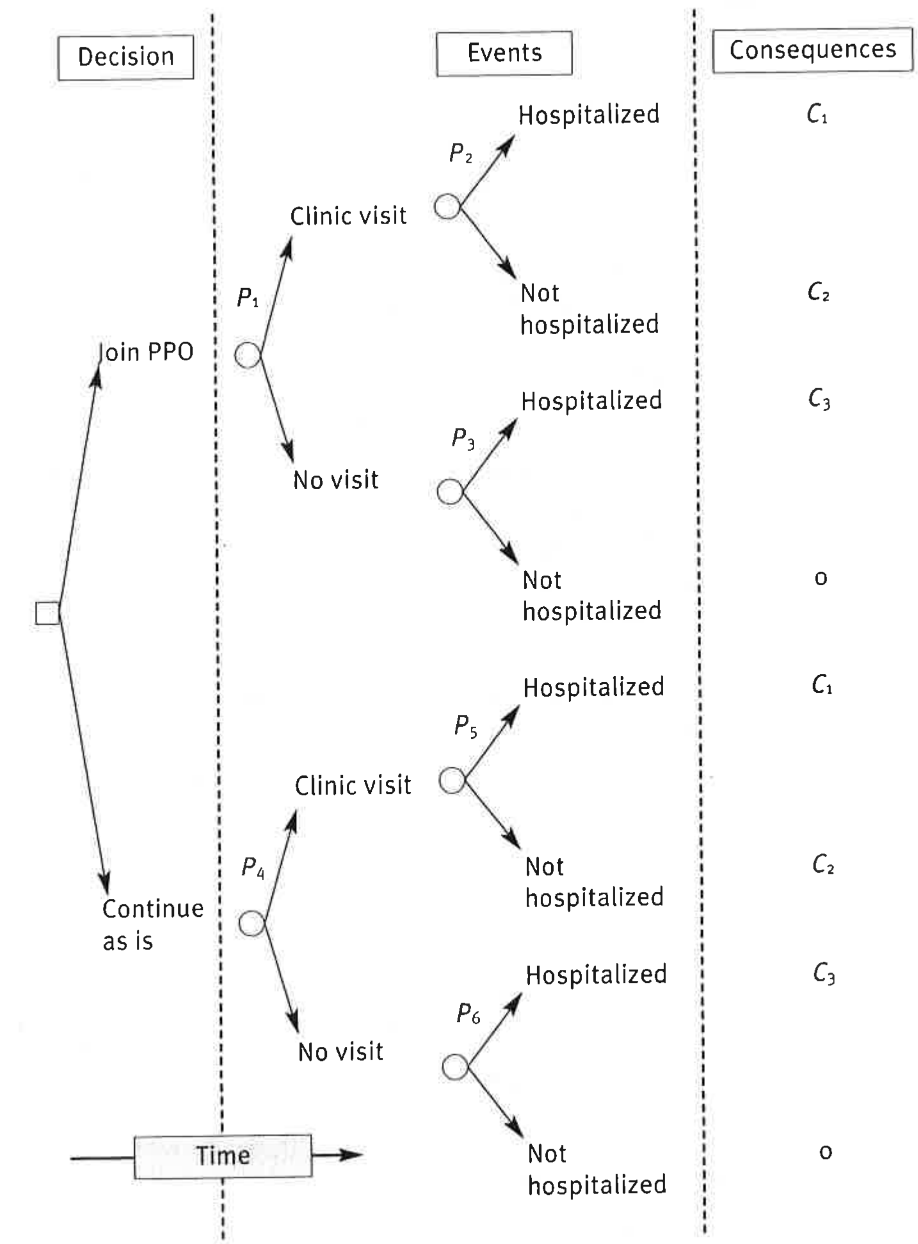


FIGURE 5.3 Consequences Are Placed to the Right of the Decision Tree

1. Decision makers are informed that a decision is looming and that they must articulate their concerns before it is completed.
2. Clients are reassured that the analysis is fair and open.
3. New insights are provided while facilitating discussion of the decision.
4. Decision makers at various levels are removed from day-to-day concerns, allowing them to ponder the impending changes. As decision makers put more thought into the decision, they develop more insight into their own beliefs.

5. Discussions among decision makers will produce further information and insights. If the analysis was done without their involvement, the positive atmosphere of collaboration would be lost.

Once a basic decision-tree structure has been organized, it is important to return to the decision makers and see if all relevant issues have been modeled. When the analyst showed Figure 5.3 to the decision makers, for example, they pointed out the following additional changes:

1. The analysis should separate general outpatient care from mental health care, because payments for the latter are capped and payments for the former are not.
2. The analysis should concentrate on employees who file claims, because only they incur costs to the employer.

The analyst revised the model to reflect these issues, and in subsequent meetings the client added still more details, particularly about the relationships among the copayment, discount, and deductible. This is important because the order in which these terms are incorporated changes the value of the different options. Negotiations between the employer and the PPO suggested that the discount is on the first dollar, before the employee pays the copayment. Employees had a $200 individual and a $500 family deductible for costs paid for clinics or hospitalization. The insurance plan required employees to meet the deductible before the copayment. Once these considerations were incorporated, the revised model presented in Figure 5.4 was created.

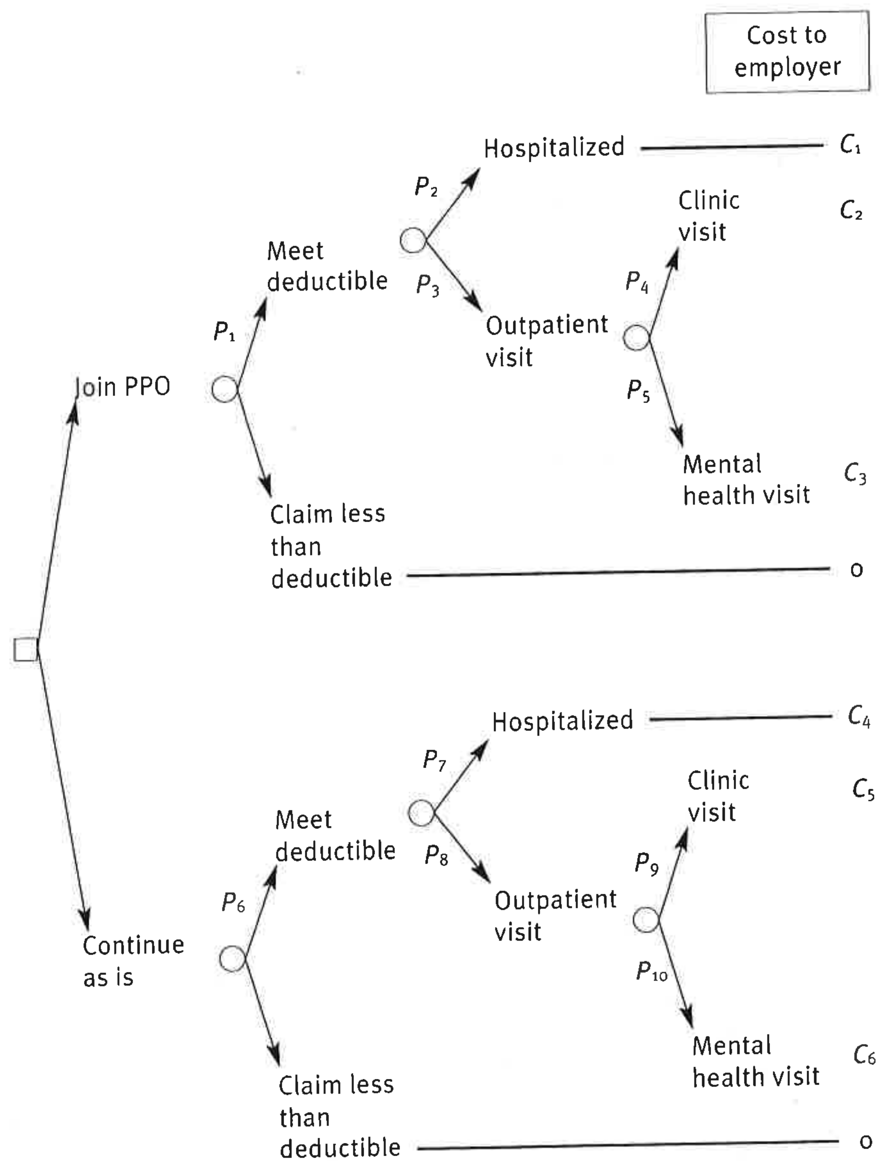
Note that for ease of presentation some nodes have been combined, and as a consequence, the sums of some probabilities may add up to more than 1.

In summary, the development of the decision tree proceeds toward increased specification and complexity. The early model is simple, later models are more sophisticated, and the final one may be too complicated to show all elements and is used primarily for analytical purposes. Each step toward increasing specification involves interaction with the decision maker—an essential element to a successful analysis.

Estimating the Probabilities

In a decision tree, each probability is conditioned on the events preceding it. Thus, P_1 in Figure 5.4 is not the probability of hospitalization but the probability of hospitalization given that the person has met the deductible

FIGURE 5.4
Revised Decision Tree



and has joined the PPO. It is important not to confuse conditional probabilities with marginal probabilities, as discussed in Chapter 3.

Conditional probabilities for the decision tree can be estimated by either analyzing objective data or obtaining subjective opinions of the experts (see Chapter 3). The probabilities needed for the lower part of the decision tree, P_4 through P_6, can be assessed by reviewing the employer's current experiences. The analyst reviewed one year of the data from the employer's records and estimated the various probabilities needed for the lower part of the decision tree.

The probabilities for the upper part of the decision tree are more difficult to assess because they require speculation regarding what might happen if employees use the preferred clinic. The decision maker identified several factors that might affect future outcomes:

1. The preferred clinic might have less efficient practices, leading to more hospitalizations and eventually more costs. The validity of this claim was examined by looking at PPO practice patterns and estimating the probabilities from these patterns.
2. Employees who join the PPO might overutilize services because they have lower copayments. If this is the case, the probabilities associated with the use of services will go up.
3. Employees moving from solo practices to group practices may lead to overutilization of specialists. Again, this will show in the probability of the utilization of services.
4. Clinicians may generate their own demand, especially when they have few visits.

To estimate the potential effect of these issues on the probability of hospitalization, the analyst reviewed the literature and brought together a panel of experts familiar with practice patterns of different clinics. The analyst asked them to assess the difference between the preferred clinic and the average clinic. The analyst then used the estimates available through the literature to assess the potential effect of these differences on utilization rates. Table 5.1 provides a summary of the synthetic estimate of what might happen to hospitalization rates by joining the PPO.

This estimate shows how the effect of joining the PPO was gauged by combining the expert's assessments with the published research literature. Although these estimates are rough, they are usually sufficient. Keep in mind that the purpose of these numbers is not to answer precisely what will happen but to determine whether one option is roughly better than the other. The assumptions made in the analysis can be tested by conducting a *sensitivity analysis*—a process in which one or two estimates are changed slightly to see if it would lead to entirely different decisions. In this example, the analysis was not sensitive to small changes in probabilities but was sensitive to the cost per hospitalization.

TABLE 5.1 Increase in Hospitalization Rates Projected at the PPO Clinic

	Difference Between PPO and Others	*Effect of 1% Change*	*Net Effect of Change*
Occupancy rate of primary hospital	−5.0%	−0.43	2.15%
Number of patients seen per day	2.50%	−0.65	−1.63%
Group versus solo practice	20.00%	0.007	0.14%
Effect of copayment reduction	+10.0%	+0.1	1.00%
Total effect of change to PPO			1.67%

Estimating Hospitalization Costs

In estimating cost per hospitalization, the analyst started with the assumption that the employees will incur the same charges as current patients at the preferred hospital. Because the provider, as a large referral center, treats patients who are extremely ill, an adjustment needed to be made. Bank employees are unlikely to be as sick, and thus will not incur equally high charges, so charges should be adjusted to reflect this difference.

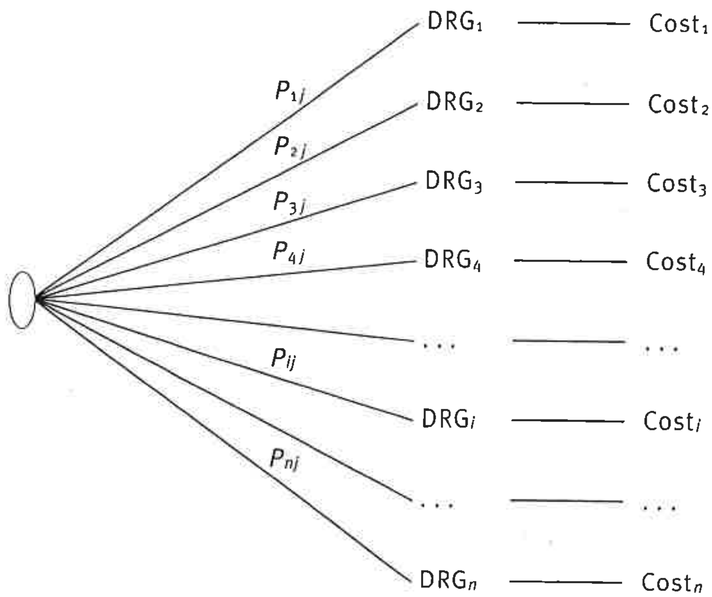
This adjustment is made by using a system developed by Medicare to measure differences in the case mix of different institutions. In this system, each group of diseases is assigned a cost relative to the average case. Patients with diseases requiring more resources have higher costs and are assigned values greater than 1. Similarly, patients with relatively inexpensive diseases receive a value less than 1. As Figure 5.5 shows, each healthcare organization is assumed to have different frequency of diseases.

The case mix for an institution is the cost of treating the disease weighted by the frequency of occurrence of the disease at that institution. Suppose Medicare has set the cost of *i*th diagnosis-related group (DRG) to be C_i, and P_{ij} measures the frequency of occurrence of DRG i at hospital j, then

$$\text{Case mix for hospital } j = \sum_{i=1,\ldots,n} P_{ij} \times C_i.$$

The ratio of two case-mix calculations at two different institutions is called a *case-mix factor*. It shows how the two institutions are different. A case-mix factor of 1 suggests that the two institutions have patients of similar diseases. Tertiary hospitals tend to have a case-mix factor that is above 1 when compared to community hospitals, indicating that tertiary hospitals see sicker patients.

FIGURE 5.5
A Decision-Tree Structure for Calculating the Case Mix at Hospital *J* Based on DRGs



To measure the cost that bank employees would have at the preferred hospital, the analyst reviewed employee records at the bank and patient records at the preferred hospital. The analyst constructed a case-mix index for each. Employees had a case-mix factor of 0.90, suggesting that these employees were not as sick as the average Medicare enrollees; the case-mix index at the preferred hospital that year was 1.17, suggesting that patients at the preferred hospital were sicker than the average Medicare patient. The ratio of the two was calculated as 1.3. This suggested that the diseases treated at the preferred hospital were about 30 percent more costly than those typically faced by employees, so the analyst proportionally adjusted the average hospitalization charges at the preferred hospital. The average hospitalization cost at the preferred hospital was $4,796. Using the case-mix difference, the analysis predicted that if bank employees were hospitalized at the preferred hospital, they would have an average hospitalization cost of $4,796 ÷ 1.3, or roughly 30 percent less cost.

Figure 5.6 shows the estimated costs for the lower and upper parts of the decision tree. Costs reported in Figure 5.6 reflect the cost per employee's family per year.

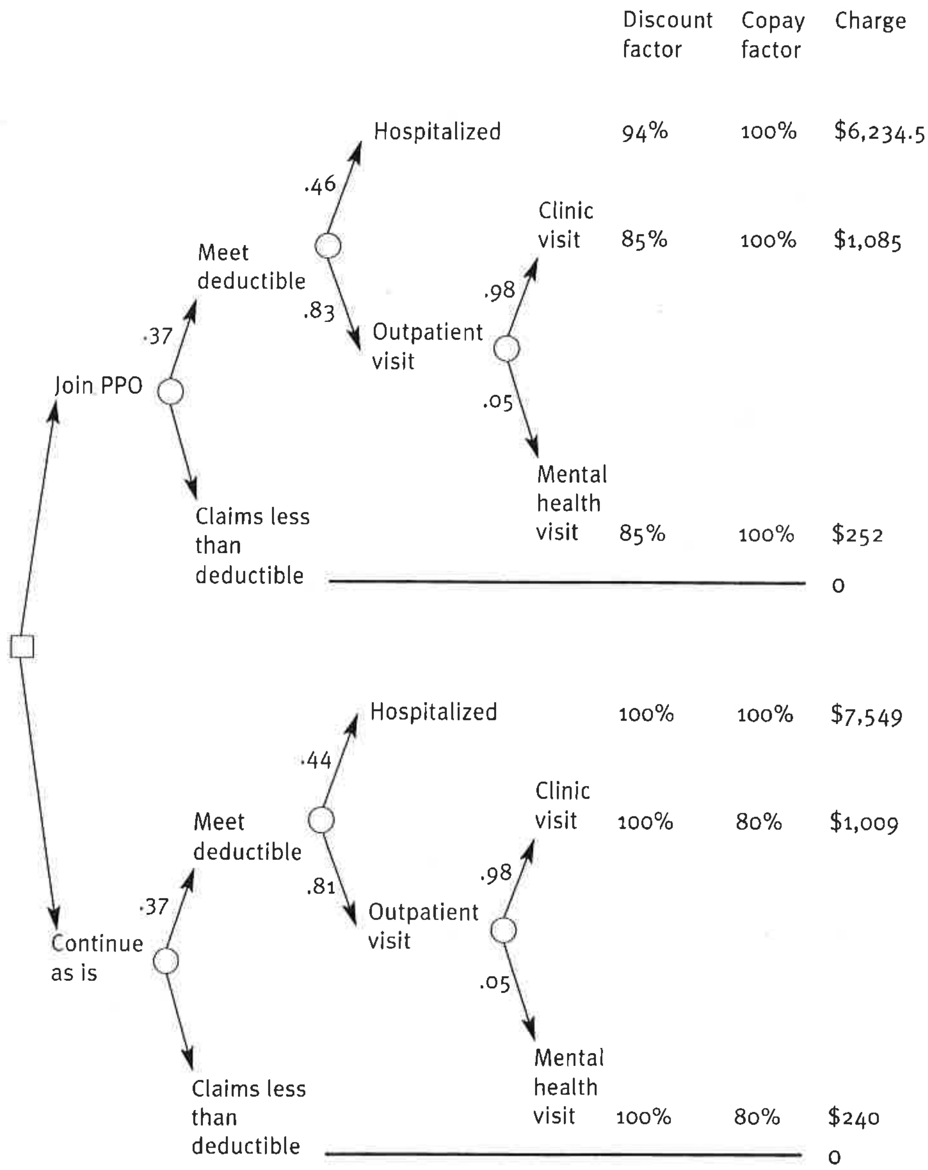


FIGURE 5.6 A Decision Tree with Estimated Costs and Probabilities Costs

Analysis of Decision Trees

The analysis of decision trees is based on the concept of expectation. The word "expectation" suggests some sort of anticipation about the future rather than an exact formula. In mathematics, the concept of *expectation* is more precise. If you believe costs $C_1, C_2, \ldots, C_n$ may happen with probabilities $P_1, P_2, \ldots, P_n$, then the mathematical expectation is

$$\text{Expected cost} = \sum_{i=1,\ldots,n} P_i \times C_i.$$

Each node of a decision tree can be replaced by its expected cost in a method called *folding back*. The expected cost at a node is the sum of the costs weighted by the probability of their occurrence. Consider, for example, the node for employees who in the current situation meet their deductibles and have outpatient visits. They have a 98 percent chance of having an outpatient visit costing $1,009 per year per person (80 percent of which is charged to the employer, and the rest of which is paid by the employee). They also have a 5 percent chance of having a mental health visit costing $240 (80 percent of which is charged to the employer, and the rest to the employee). The expected cost to the employer for outpatient visits per employee per year is then calculated as

$$\text{Employer's expected cost for outpatient visits} = (0.98 \times 0.80 \times \$1{,}009) + (0.05 \times 0.80 \times \$240) = \$800.66.$$

This expected cost can replace the node for outpatient visits in the "continue as is" situation in the decision tree. Likewise, the process can now be repeated to fold back the decision tree further and replace each node with its expected cost. Figure 5.7 shows the calculation of the expected cost of continuing as is through three steps.

The employer's expected cost for joining the PPO was calculated to be $871.08 per employee's family per year. This is $597.86 per employee's family per year less than the current situation. As the firm has 992 employees, the analysis suggests that switching to the PPO will result in cost savings of almost half million dollars per year.

The problem with the folding-back method is that it is not easy to represent the calculation in formulas with programs such as Excel; too much of the information is visual. However, there is another way of analyzing a decision tree that takes advantage of the decision tree's structure and does not require folding-back procedures. First, all the probabilities for each path in the decision tree are multiplied together to find the joint probability of the path. For example, after joining the PPO, the joint probability of meeting the deductible and being hospitalized is provided by multiplying 0.37 (the probability of meeting the deductible for people who join the PPO) by 0.46 (the probability of being hospitalized if the person has joined the PPO and meets the deductible). To calculate the expected cost/value, the joint probability of each path is multiplied by the corresponding cost/value and summed for each option. Table 5.2 shows each

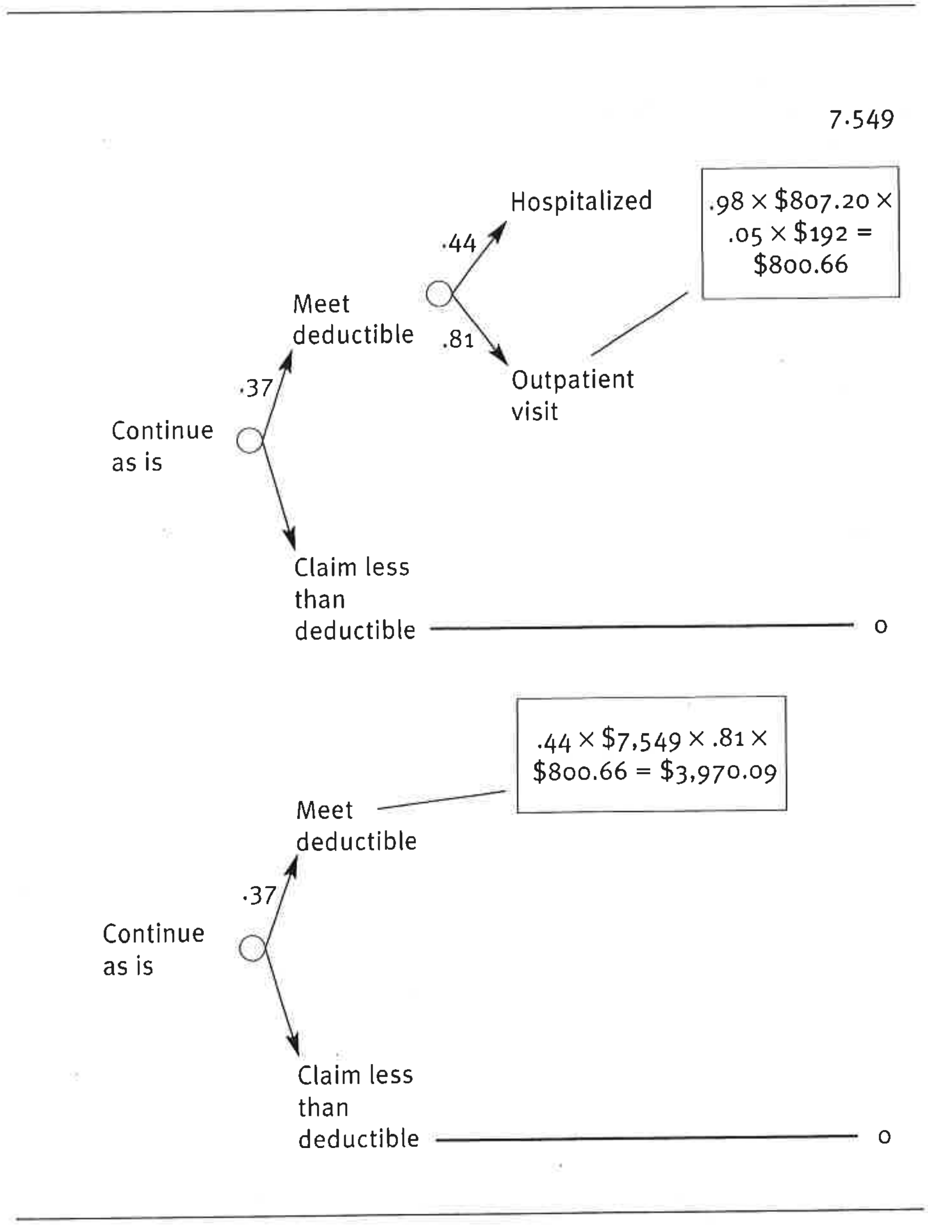


FIGURE 5.7 Starting from the Right, Each Node Is Replaced with its Expected Value

of the paths in the upper part of the decision tree in Figure 5.7, the corresponding joint probability of the sequence, and its associated costs.

This information can be used to calculate the expected cost by multiplying the cost and the probability of each path and summing the results. Using the terms introduced in Figure 5.4, the expected cost of joining the PPO can be calculated using the following formula:

TABLE 5.2 Using the Paths in a Decision Tree to Calculate Expected Cost

Path Depicting the Sequence of Events After Joining the PPO	*Formula Using Terms in Figure 5.4*	*Joint Probability of Sequence*	*Cost of Sequence*	*Probability of Sequence Times its Cost*
Meet deductible, Hospitalization	P_1P_2	0.17	\$3,467.88	\$590.23
Meet deductible, Outpatient visit, Clinic visit	$P_1P_3P_4$	0.30	\$922.25	\$277.56
Meet deductible, Outpatient visit, Mental health visit	$P_1P_3P_5$	0.02	\$214.20	\$3.29
Expected cost of joining PPO				\$871.08

$$\text{Expected cost (Joining PPO)} = (P_1 \times P_2 \times C_1) + (P_1 \times P_3 \times P_4 \times C_2) + (P_1 \times P_3 \times P_5 \times C_3).$$

Note that the above formula does not show situations that lead to zero cost (not meeting the deductible or meeting the deductible but not having any additional healthcare utilization). If you wanted to show the path leading to zero cost for employees who do not meet the deductible, you would have to add the following term to the above formula: $+ (1 - P_1) \times 0$.

The expected cost of continuing as is can be calculated as

$$\text{Expected cost (Continuing as is)} = (P_6 \times P_7 \times C_4) + (P_6 \times P_8 \times P_9 \times C_5) + (P_1 \times P_6 \times P_8 \times C_6).$$

When expected cost is expressed as a formula, it is possible to enter the formula into Excel and ask a series of "what if" questions. The analyst can change the values of one variable and see the effect of the change on the expected-cost calculations.

Sensitivity Analysis

Some analysts mistakenly stop the analysis after a preferred option has been identified. This is not the point to end the analysis but the start of real understanding of what leads to the choice of one option over another. As previously mentioned, the purpose of an analysis is to provide insight and not to produce numbers. One way to help decision makers better understand the structure of their decision is to conduct a sensitivity analysis on the data to see if the conclusions are particularly sensitive to some inputs. Many decision makers are skeptical of the numbers used in the analysis and wonder if the conclusions could be different if the estimated numbers were different. *Sensitivity analysis* is the process of changing the input parameters until the output (the conclusions) is affected. In other words, it is the process of changing the numbers until the analysis falls apart and the conclusions are reversed.

Sensitivity analysis starts with changing a single estimate at a time until the conclusions are reversed. Two point estimates are calculated, one for the best possible scenario and the other for the worst possible scenario. For example, in estimating the costs associated with joining the PPO, the probability of hospitalization given that the person has met the deductible is an important estimate about which the decision maker may express reservations. To understand the sensitivity of the conclusions to this probability, three estimates are obtained: (1) the probability set to maximum (when everyone is hospitalized), (2) the probability set to minimum (when no one is hospitalized), and (3) the initial expected value, or the *base case* (see Table 5.3).

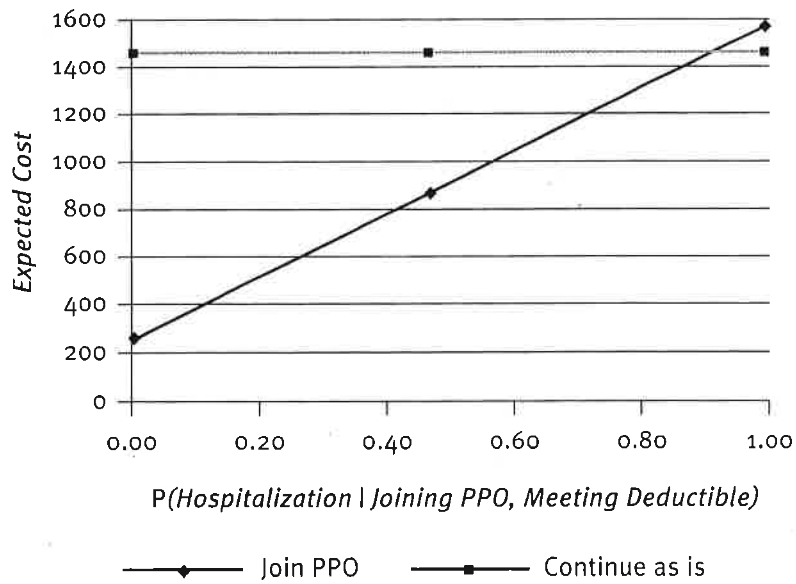
Changing this conditional probability leads to a change in the expected cost of joining the PPO. To understand whether the conclusions are sensitive to the changes in this conditional probability, analysts typically plot the changes. The x-axis will show the changing estimate—in this case, the conditional probability of hospitalization for employees who meet the deductible and have joined the PPO. The y-axis shows the value of the decision options—in this case, either the expected cost of joining the PPO or the expected cost of continuing as is. A line is drawn for each option. Figure 5.8 shows the resulting sensitivity graph.

Note that in Figure 5.8, joining the PPO is preferred, for the most part, to continuing as is. Only at very high probabilities of hospitalization, which the decision maker might consider improbable, is the situation reversed. The conclusions are reversed at 0.93, which is called the *reversal*, or *break-even, point.* If the estimate is near the reversal point, then the

TABLE 5.3 Effect of Changing the Probability of Hospitalization

	Probability	*Join PPO*	*Continue As Is*
Maximum	1.00	$1563.96	$1,468.93
Estimated	0.46	$871.08	$1,468.93
Minimum	0.00	$280.85	$1,468.93

FIGURE 5.8
Sensitivity of Conclusions to Conditional Probability of Hospitalization



analyst will be concerned. If the estimate is far away, the analyst will be less concerned. The distance between the current estimate of 0.37 and the reversal point of 0.93 suggests that small inaccuracies in the estimation of the probability will not matter in the final analysis. What will matter? The answer can be found by conducting a sensitivity analysis on each of the parameters in the analysis to find one in which small changes will lead to decision reversals.

The analysis calculates the cost of hospitalization for employees from current hospital costs at the preferred hospital. PPO hospitalization costs are reduced by a factor of 1.3 to reflect the case mix of the employed population. It assumes that the employed population is less severely sick than the general population at the preferred hospital. The reversal point for the estimate of case mix is 0.65, at which point the cost of hospitalization for the employees at the preferred hospital would be estimated to be $6,935. This seemed unlikely, as it would have claimed that the bank's employees needed to be significantly sicker than the current patients at the preferred hospital, which is a national referral center.

It is important to find the reversal points for each of the estimates in a decision tree. This can be done by solving an equation in which the variable of interest is changed to produce an expected value equal to the alternative option. This can also be done easily in Excel using the goal-seeking tool, which finds an estimate for the variable of interest that would make the difference between the two options become zero.

So far, changing one estimate to see if it leads to a decision reversal has been discussed. What if analysts change two or more estimates simultaneously? How can the sensitivity of the decision to changes in multiple estimates be examined? For example, what if both the estimates of probability of hospitalization and the cost of hospitalization were wrong? To assess the sensitivity of the conclusions to simultaneous changes in several estimates, the technique of ***linear programming*** must be used. This technique allows the minimization of an objective subject to constraints on several variables. The absolute difference between the expected value of the two options is minimized subject to constraints imposed by the low and high ranges of the various estimates.

For example, you might minimize the difference between the expected cost of joining the PPO and continuing as is subject to a case mix ranging from 1 to 1.4 and the conditional probability of hospitalization ranging from 0.3 to 0.5. Mathematically, this shown as follows:

Minimize objective = Expected cost (Joining HMO) – Expected cost (Continue as is),

where

- $1 <$ Case mix of PPO to bank employees < 1.4; and
- $0.3 < P(\text{Hospitalization}|\text{Joining PPO, Meeting deductible}) < 0.5$.

It is difficult to solve linear programs by hand. One possibility is to solve for the worst-case scenario. In this situation, the assumption is that joining the PPO will lead to the worst rate of hospitalization (0.5) and the worst cost of hospitalization (case mix of 1). Even in this worst-case scenario, joining the PPO remains the preferred option. In addition to using the worst-case scenarios, it is also possible to use Excel's solver tool as a relatively easy way to use linear programming. Using the solver tool, the analyst found that there are no solutions that would make the difference between joining the PPO and continuing as is become zero subject to the above constraints. Therefore, even with both constraints changing at the same time, there is no reversal of conclusions.

Missed Perspectives

The purpose of any analysis is to provide insight. Often when decision makers review an analysis, they find that important issues are missed. In the

PPO example, one decision maker believed the potential savings were insufficient to counterbalance the political and economic costs of instituting the proposed change. The current healthcare providers were customers of the client, and signing a contract with the PPO might alienate them and induce them to take their business elsewhere. Incorporating the risk of losing customers would improve the calculations and help the bank decide whether the savings would counterbalance the political costs (more on this in the next section).

Furthermore, additional discussion leads to another critical perspective: Would it be better to wait for a better offer from a different provider? The consequences of waiting could have been incorporated into the analysis by placing an additional branch from the decision node, marked as "wait for additional options." This would have provided a more comprehensive analysis of the decision.

New avenues often open when an analysis is completed. It is important to remember that one purpose of analysis is to help decision makers understand the components of their problem and to devise increasingly imaginative solutions to it. Therefore, there is no reason to act defensively if a client begins articulating new options and considerations while the analyst presents the findings. Instead, the analyst should actively encourage the clients to discuss their concerns and consider modifying the analysis to include them.

A serious shortcoming with decision trees is that many clients believe they show every possible option. Actually, there is considerable danger in assuming that the problem is as simple as a decision tree makes it seem. In this example, many other options may exist for reducing healthcare costs aside from joining the PPO; but perhaps because they were not included in the analysis, they will be ignored by the decision maker, who (like the rest of us) is victim to the "out of sight, out of mind" fallacy (Silvera et al. 2005; Fischhoff, Slovic, and Lichtenstein 1978).

The "myth of analysis" can explain why things not seen are not considered. This myth is the belief that analysis is impartial and rests on proper assumptions and that it is robust and comprehensive. Perpetuating this myth prevents further inquiry and imaginative solutions to problems. Decision trees could easily fall into this trap, because they appear so comprehensive and logical that decision makers fail to imagine any course of action not explicitly included in them.

The final presentation of a decision-tree analysis is broken into two segments. First, the report summarizes the results of the decision tree and the sensitivity analysis. The report of the analysis should have these examinations in the appendix and the base-case, best-case, and worst-case solutions in the main report. The section containing the recommendation of the reports should refer to the sensitivity of the conclusions to changes in the input. Second, the analyst should ask the clients to share their ideas about options and considerations not modeled in the analysis. If one does not explicitly search for new alternatives, the analysis might do more harm than good. Instead of fostering creativity, it can allow the analyst and decision maker to hide behind a cloak of missed options and poorly comprehended mathematics.

Expected Value or Utility

Sometimes the consequences of an event are not just additional cost or savings, and it is important to measure the utility associated with various outcomes. In these circumstances, one measures the value of each consequence in terms of its utility and not merely its costs. To fold back the decision tree, expected utility is used instead of the expected cost.

When Bernoulli (1738) was experimenting with the notion of expectation, he noticed that people did not prefer the alternative with the highest expected monetary value; people are not willing to pay a large amount of money for a gamble with infinite expected return. In explanation, Bernoulli suggested that people maximize utility rather than monetary value, and costs should be transformed to utilities before expectations are taken. He named this model *expected utility*.

According to expected utility, if an alternative has n outcomes with costs $C_1, \ldots, C_n$ associated probabilities of $P_1, \ldots, P_n$, and if each cost has a particular utility to the decision maker of $U_1, \ldots, U_n$, then

$$\text{Expected utility} = \textstyle\sum_{i=1,\ldots,n} P_i \times U_i.$$

Bernoulli resolved the paradox of why people would not participate in a gamble with infinite return by arguing that the first dollar gained has a greater utility than the millionth dollar. The beauty of a utility model is that it allows the marginal value of gains and losses to decrease with their magnitude. In contrast, mathematical expectation assigns every dollar the same value. When the costs of outcomes differ considerably—say, when one outcome costs \$1,000,000 and another \$1,000—one can prevent small gains from being overvalued by using utilities instead of costs.

Utilities are also better than costs in testing whether benefits meet the client's goals. Using costs in the PPO analysis, joining the PPO would lead to expected savings of about half a million dollars. Yet, when the bank had not acted six months after completing the analysis, it became clear that this savings was not sufficient to cause a change because nonmonetary issues were involved. The analyst could have uncovered this problem if, instead of monetary returns, he had used utility estimates.

Chapter 2 describes how to measure utility over many dimensions, both monetary and nonmonetary, was described. In the PPO example, cost was not the sole concern—the bank had many objectives for changing its healthcare plan. If it wanted only to lower costs, it could have ceased providing healthcare coverage entirely, or it could have increased the copayment. The bank was concerned about the employees' reactions, which it anticipated would be based on concerns for quality, accessibility, and, to a lesser extent, cost to employees. A utility model should have been constructed for these concerns, and the model should have been used to assess the value of each consequence.

Utility is also preferable for clients who must consider attitudes toward risk. This is because expected utility, in contrast to expected cost, reflects attitudes toward risk. A risk-neutral individual bets the expected monetary value of a gamble. A risk taker bets more on the same gamble because she associates more utility to the high returns. A risk-adverse individual cares less for the high returns and bets less. Research shows that most individuals are risk seeking when they can choose between a small loss and a gamble for a large gain, and they are risk adverse when they must choose between a small gain and a gamble for a large loss (Kahneman and Tversky 1979).

A client, especially when trying to decide for an organization, may exclude personal attitudes about risk and request that the analysis of the decision tree be based on expected cost and not expected utility. Thus, the client may prefer to assume a risk-neutral position and behave as if every dollar of gain or loss were equivalent. The advantage of making the risk attitudes explicit is that it leads to insights about one's own policies; the disadvantage is that such policies may not be relevant to other decision makers.

Transformation of costs to values/utilities is important in most situations. But when the analysis is not done for a specific decision maker, monetary values are paramount, the marginal value of a dollar seems constant across the range of consequences, and attitudes toward risk seem irrelevant, then it may be reasonable to explicitly measure the cost and implicitly consider the nonmonetary issues.

When asked, the executives raised the issue that money was not the sole consideration. Many of the hospitals' CEOs were on the bank's executive board. The bank was concerned that by preferring one healthcare provider, they may lose their goodwill and, at an extreme, the current providers may shift their funds to a competing bank. Figure 5.9 shows the resulting dilemma faced by the bank.

Figure 5.9 shows that the bank faces the loss of half a million dollars per year for continuing as is. Alternatively, it can join the PPO but faces a chance of losing its healthcare customer's goodwill. To further analyze this decision tree, it is necessary to assess the probability of losing the healthcare organization's goodwill and the value or utility associated with the overall affect of both cost and goodwill. If attitudes toward risk do not matter in this analysis, then the analyst can focus on measuring overall value using MAV models discussed in Chapter 2. Assume that goodwill is given a weight of 0.75, and cost savings of half a million dollars per year is given a weight of 0.25. Also assume that the probability of healthcare organizations shifting their funds to other banks is considered to be small—say, 1 percent. Figure 5.10 summarizes these data.

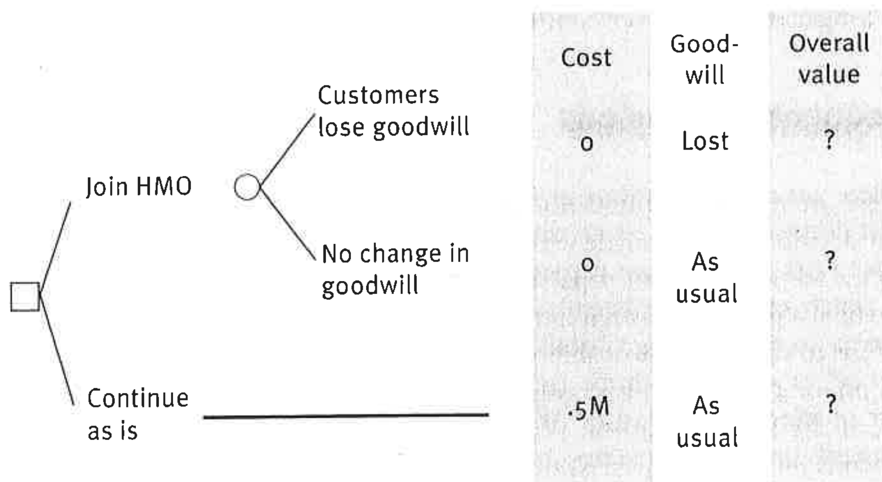


FIGURE 5.9 A Decision Tree Showing Bank's Concern over Losing Healthcare Customers

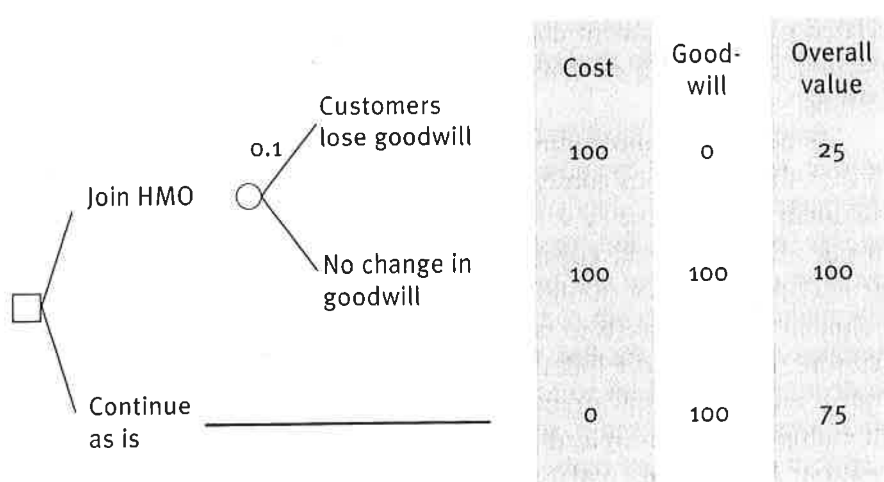


FIGURE 5.10 Value Associated with Cost and Losing Goodwill

The expected value for joining the PPO can be calculated by folding back, starting with the top right side. There is a probability of 1 percent of having an overall value of 25 versus a probability of 99 percent of having a value of 100. The expected value for this node is $(0.01 \times 25) + (0.99 \times 100) = 99.25$. The expected value for the continuing as is node is shown as 75. Therefore, despite the small risk of losing goodwill with some healthcare bank customers, the preferred course of action is to go ahead with the change.

The analyst can conduct a sensitivity analysis to see at what probability of losing goodwill is joining the PPO no longer reasonable (see Figure 5.11). As the probability of losing goodwill increases, the value of joining the PPO decreases. At probabilities higher than 0.35, joining the PPO is no longer preferred over continuing as is.

Sequential Decisions

There are many situations in which one decision leads to another. A current decision must be made with future options in mind. For example, consider a risk-management department inside a hospital. After a sentinel event in which a patient has been hurt, the risk manager can step in with several actions to reduce the probability of a lawsuit. The patient's bill can be written off, or a nurse might be assigned to stay with the patient for the remainder of the hospitalization. Whether the risk manager takes these steps depends on the effectiveness of the preventive strategy. It also depends on what the hospital will do if it is sued. For example, if sued, the risk manager faces the decision to settle out of court or to wait for the verdict. Thus, the two decisions are related. The first decision of preventing the lawsuit is related to the subsequent decision of the disposition of the lawsuit after it occurs. This section describes how to model and analyze interrelated decisions.

As before, the most immediate decision is put to the left of a decision tree, followed by its consequences to the right. If there are any related subsequent decisions, they are entered as nodes to the right of the decision tree, or after specific consequences. For example, the decision to prevent lawsuits is put to the left in Figure 5.12. If the lawsuit occurs, a subsequent decision needs to be made about what to do about the lawsuit. Therefore, following the link that indicates the occurrence of the lawsuit, a node is entered for how to manage the lawsuit. To analyze a decision tree with multiple decisions in it, the folding back process is used with one new exception: All nodes are replaced with their expected value/cost as before,

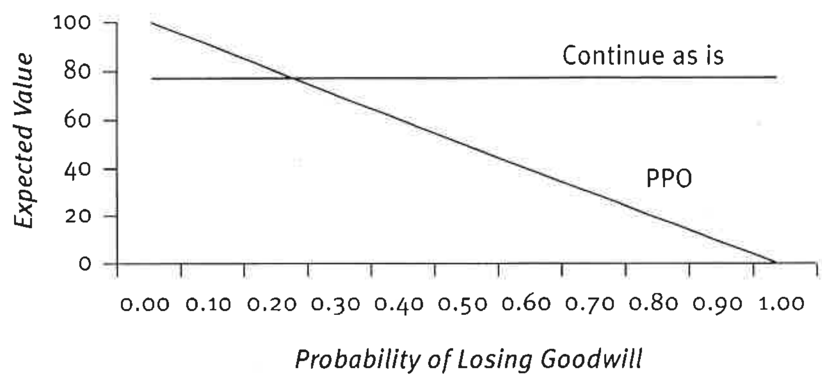


FIGURE 5.11 Sensitivity of Conclusions to Probability of Losing Goodwill After Joining the PPO

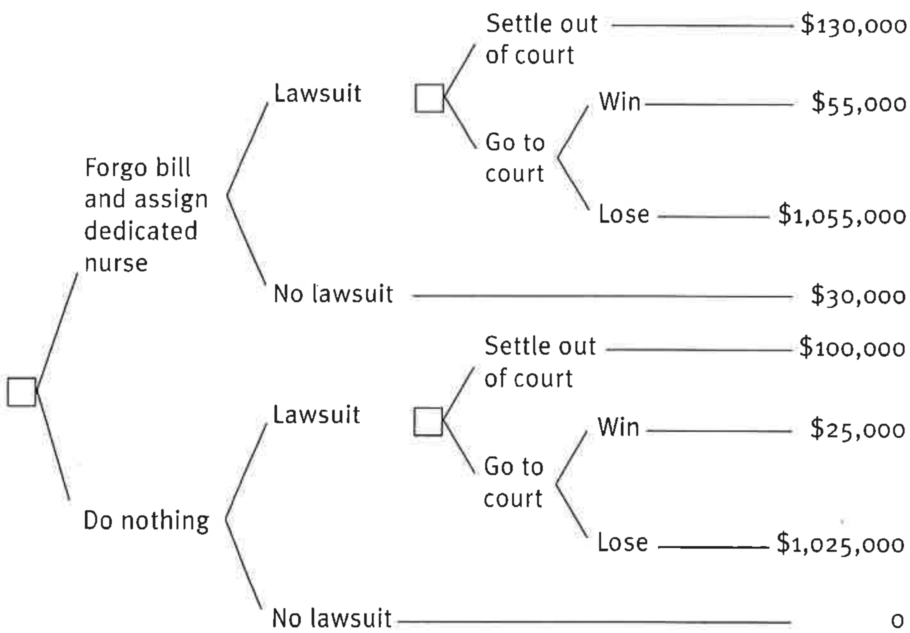
but the decision node is replaced with the minimum cost or maximum utility/value of the options available at that node. This is done because, at any decision node, the decision maker is expected to maximize his value/utilities or minimize cost.

In the book *Quick Analysis for Busy Decision Makers*, Behn and Vaupel (1982) suggest how decision-tree analysis can be applied to the problem of settling out of court. Here, their suggestions are applied to a potential malpractice situation. As Figure 5.12 shows, the cost of forgoing the hospital bill and assigning a dedicated nurse to the patient is estimated at $30,000. If the case is taken to the court, there is an estimated $25,000 legal cost. If the hospital loses the case, the verdict is assumed to be for $1 million. Figure 5.12 summarizes these costs. The question is whether it is reasonable to proceed with the preventive action. To answer this question, three probabilities are needed:

1. The probability that the person will file a lawsuit if no preventive action is taken.
2. The probability of a lawsuit if preventive action is taken.
3. The probability of a favorable verdict if the case goes to court.

The estimation of these probabilities and the cost payments need to be appropriate to the situation at hand. Figure 5.12 provides rough estimates for these probabilities and costs, but in reality the situation should be tailored to the nature of the patient, the injury, and experiences with such lawsuits. The published data in the literature can be used to tailor the analysis to the situation at hand. Following are some examples of where the numbers might come from:

FIGURE 5.12
The Decision to Prevent a Malpractice Lawsuit



- Driver and Alemi (1995) provide an example of estimating probability of lawsuits from patients' characteristics and circumstances surrounding the incidence. They built a Bayesian probability model for predicting whether the patient will sue from data such as the patient's age, gender, family income, and length of relationship with the doctor; the severity of injury; the patient's attribution of cause of the event; the number of mishaps; the patient's legal or healthcare work experience; and the type of sentinel event.
- Selbst, Friedman, and Singh (2005) provide objective data on epidemiology and etiology of lawsuits involving children. They show that, in 1997, hospitals settled in 93 percent of cases involving mostly diagnostic errors in emergency departments for meningitis, appendicitis, arm fracture, and testicular torsion. Among the costs not settled, the courts found in favor of the hospital in 80 percent of cases and in favor of the patient in 20 percent of cases. The payout depended on the nature of injury. In 1997, average payout was $7,000 for emotional injury, $149,000 for death of the patient, $300,000 for major permanent injury, and $540,000 for quadriplegic injury.
- Bors-Koefoed and colleagues (1998) provide statistical models for assessing the probability of an unfavorable outcome for a lawsuit and the likely amount of payout for obstetrical claims. They showed that

> Indicators of increased indemnity payment were: non-reassuring intrapartum fetal heart rate tracing, later year of delivery, intensity of long-term care required, and participation of a particular defense law firm. Perinatal or childhood death, the use of pitocin, and settlement date increasingly removed from the occurrence date were the determinants of decreased payments in this model. Finally, the presence of major neurological deficits, the prolongation of a case, and the involvement of multiple law firms and defense witnesses increased the expense charged to and paid by the insurance company.

Many similar articles exist in the Medline literature from which both the maximum payout and the probability of these payouts can be assessed. If the probability of winning in court for the case at hand is 60 percent, and the probability of lawsuit is 15 percent and is reduced to 5 percent after the preventive action, then the optimal decision under these assumptions can be calculated.

To fold back the decision tree, start from the top right side and first fold back the node associated with the court outcomes. The expected cost for going to the court is (0.6 × \$55,000) + (0.4 × \$1,055,000) = \$455,000. At this point, the decision tree is pruned as shown in Figure 5.13.

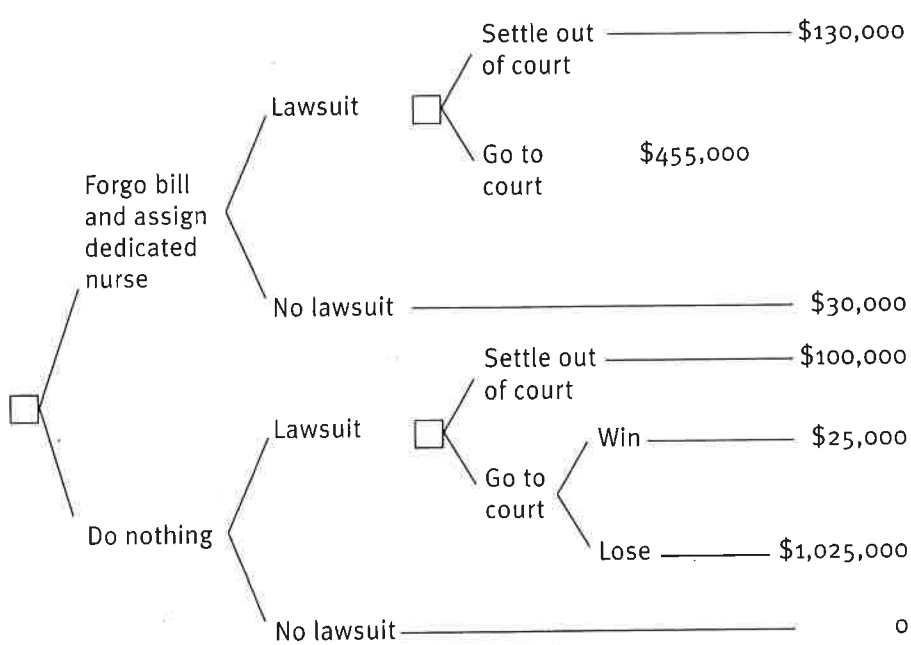


FIGURE 5.13 Replacing the Court Outcomes with Their Expected Costs

Settlement out of court will cost \$130,000 and is preferred to going to court. Therefore, the expected cost for a lawsuit is \$130,000. Note that in a decision node, always use the minimum expected cost associated with the options available at that node. This now reduces the decision tree as shown in Figure 5.14.

Next, calculate the expected cost for preventive action as $(0.05 \times \$130,000) + (0.95 \times \$30,000) = \$35,000$. This results in the final decision tree shown in Figure 5.15.

FIGURE 5.14 The Expected Cost for the Decison Node Is the Option with Minimum Cost

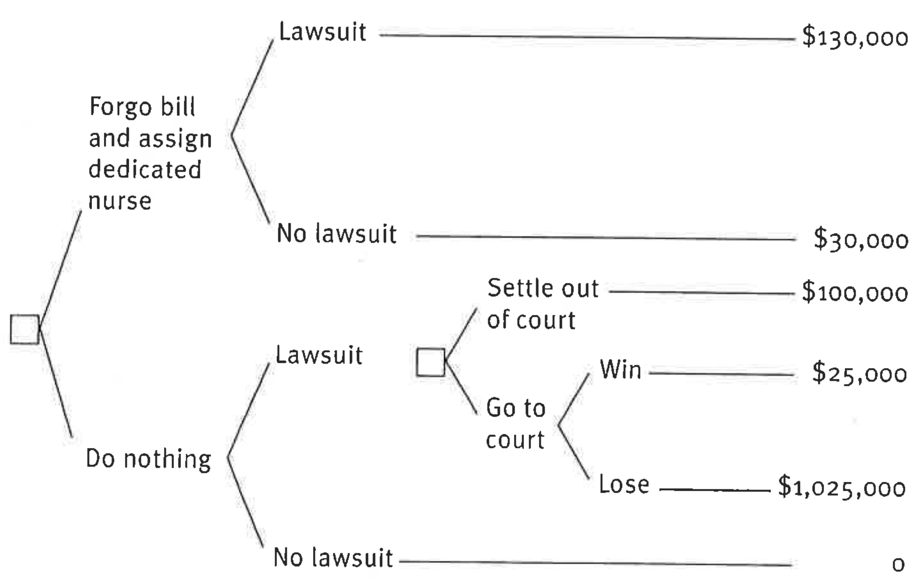
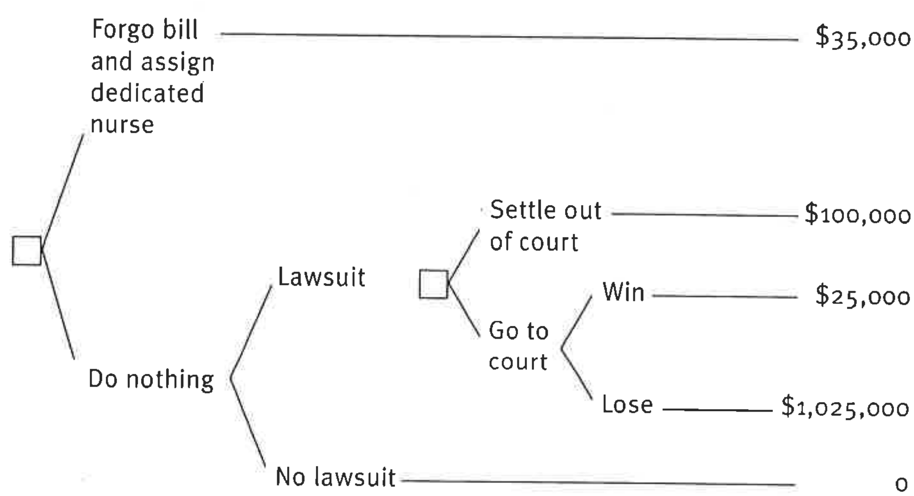


FIGURE 5.15 The Expected Cost of Preventing a Lawsuit



A similar set of calculations can be carried out for the option of doing nothing. In these circumstances, the expected cost associated with the court case is $(0.6 \times \$25{,}000) + (0.4 \times \$1{,}025{,}000) = \$425{,}000$. The preferred option is to settle out of court for $100,000. The expected cost for a lawsuit is $100,000. The expected cost for doing nothing is $(0.15 \times \$100{,}000) + (0.85 \times 0) = \$15{,}000$, which is lower than the expected cost of taking preventive action. For this situation (given the probabilities and costs estimated) the best course of action is to not take any preventive action. A sensitivity analysis can help find the probabilities and costs at which point conclusions are reversed.

Summary

Previous chapters have presented several useful tools a decision analyst can use in modeling decisions, such as how to quantify the values of stakeholders and how to systematically include a consideration of uncertain events in making decisions. This chapter presents another tool for modeling decisions—decision trees. Decision trees are useful in situations when making a decision is dependent on a series of events occurring. These situations include both subjective value and uncertainty, and decision trees are able to accommodate such simultaneous considerations in making decisions. A decision tree models a temporal sequence of events, beginning with the root, which represents a particular decision. Chance nodes emanate from the root of the decision tree and represent all the possible events that follow from a given decision. The final element of a decision tree is the consequences, or the potential effects or results of the various chance nodes.

A decision tree, once constructed, can provide decision makers with a preferred option. This is done by calculating an expected value through the folding back of the tree, where each node is replaced with its expected value. The decision-making process that utilizes decision trees does not end with the identification of a preferred option. Sensitivity analysis is done to see if conclusions can be changed with minor changes in the estimates.

Review What You Know

1. Define the following terms.
   a. Decision node

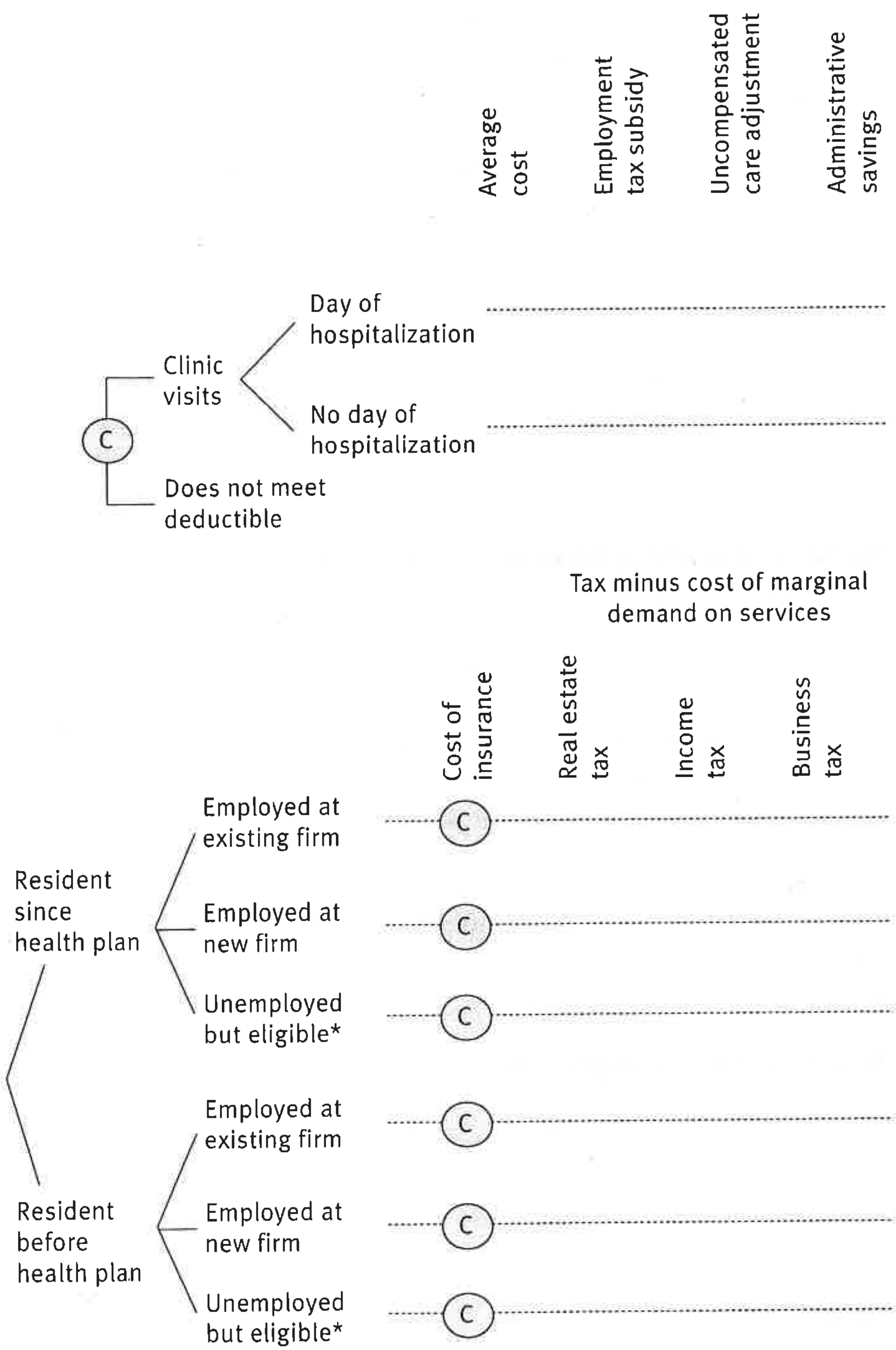
   b. Event node
   c. Decision tree
   d. Sensitivity analysis
   e. Folding back
   f. Expected cost
   g. Expected value
   h. Sequential decisions
2. What is the expected cost in a gamble that has a 10 percent chance of losing $10,000? Draw the node and calculate the expected cost.
3. Recalculate the expected cost of assigning a dedicated nurse as a preventive action if instead of the potential loss of $1,055,000 the maximum award was $250,000.
4. Recalculate the expected cost of joining the PPO if hospitalization rate for the preferred hospital was underestimated and the correct rate was 0.54 rather than 0.46.
5. When the probabilities of arcs coming out of a node do not add up to 1, what does this mean in terms of the existence of mutually exclusive and exhaustive events?
6. Does calculating case-mix index involve calculating an expected value? What events and which probabilities are involved in this calculation?
7. What is a risk-averse person?
8. Using the decision tree for joining the PPO, calculate the probability of each pathway that comes out of the node for doing nothing.

Rapid-Analysis Exercises

Option 1: Analyzing Local Universal Health Insurance

Evaluate what will happen if your county chooses to self-insure all residents, both employed and unemployed, who have lived in the county for at least two years. The worksheet shown in Figure 5.16 supplies a decision tree to help you analyze this decision. The premiums for the new plan (node C) can be estimated as the current costs of hospital and clinic services minus the following factors: (1) administrative healthcare costs go down in single payer systems; (2) uncompensated care costs are reduced when almost everyone is insured; and (3) the federal government will pay the equivalent of the current employer's tax subsidy to the county. Use the first part of Figure 5.16 to determine the estimated premiums for the new plan. Then, use these premiums as input on the second part of Figure 5.16, which evaluates the plan's impact on taxes.

FIGURE 5.16
Worksheet for Analyzing Local Universal Health Insurance



* Residents are eligible for coverage after they have lived in the country for at least two years.

When a region has health insurance for every resident, employers' costs of insurance will be lower. Therefore, more employers will move to the county to take advantage of these savings. These new employers will pay a business tax, thus enhancing the tax base of the county. New employers need new employees, so more residents will move to the county. These new residents will contribute to the county's income tax and will pay real estate taxes. Offering free health insurance to all residents will also attract the unemployed and the chronically sick, though they may be deterred by a required waiting period of two years. Use the second part of the decision tree in Figure 5.16 to determine these costs.

In your analysis, be sure to calculate the impact of this health insurance plan as per resident, not per business, per year. Include data from literature or from knowledgeable experts in your analysis. Multiply the impact of the program (per resident) times the projected number of residents to calculate the total costs of or the savings associated with the program.

Option 2: Analyzing Fetal and Maternal Rights

A clinician is facing an important dilemma of choosing between fetal and maternal rights. The patient is a 34-year-old woman with a 41-week intrauterine pregnancy. The mother is refusing induction of labor. Without the labor induction, the fetus may die. Despite this risk, the mother wants to pursue a vaginal delivery. What should the clinician do?

Model the clinician's decision when the mother refuses to undergo a necessary life-saving cesarean for the infant. Make sure that your analysis is based on the viability of the infant as well as the intrusiveness of the clinician's intervention. Create a decision tree and solicit the utility of various courses of action under different probabilities (see Mohaupt and Sharma 1998).

Audio/Visual Chapter Aids

To help you understand the concepts of decision trees, visit this book's companion web site at ache.org/DecisionAnalysis, go to Chapter 5, and view the audio/visual chapter aids.

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