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AN ECONOMIC DESIGN OF TUKEY'S CONTROL CHART

Chau-Chen Torng [a] , Pei-Hsi Lee [a] & Chun-Chieh Tseng [a]
[a] Graduate School of Industrial Engineering and Management National Yunlin University of Science and Technology, 123 University Road, Section 3, Douliou, Yunlin, 64002, Taiwan, R.O.C.
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Chau-Chen Torng, Pei-Hsi Lee* and Chun-Chieh Tseng
Graduate School of Industrial Engineering and Management
National Yunlin University of Science and Technology
123 University Road, Section 3, Douliou, Yunlin 64002, Taiwan, R.O.C.

ABSTRACT

The significant features of Tukey's control chart include characteristics such as: easy setup, use of single observation to monitor process, and suitability to monitor destructive testing processes. The control limit width and sampling interval should be determined before Tukey's control chart can be used. In this study, Duncan's cost function is modified to construct an economic design model of Tukey's chart and to attain optimal design. Furthermore, methods of calculating error probabilities of Tukey's chart under normal conditions are constructed and applied in the economic design model. A real-world example of IC packaging illustrates the application of Tukey's chart to economics. The results of sensitivity analyses demonstrate that shorter in-control time or larger process variation cost incurs higher control cost.

Keywords: Tukey's control chart, economic design, inter-quartile range, box plot

1. INTRODUCTION

When an assignable cause occurs during the manufacturing process, process variation will occur and result in defective products. Using the control chart technique, process variation can easily be detected by staff, and then swiftly repaired so as to reduce the amount of defective product.

Currently, there are many types of control charts; when process is monitored, appropriate control charts must be selected according to sampling methods and monitoring procedures. Many electronics manufacturers utilize a destructive testing approach to measure the observations of process. After destructive testing and inspection, the sample is destroyed and cannot be sold on the market. Generally, for the process monitoring of this type of product, only one sample is taken to measure the observation so as to reduce cost. In this way, most destructive testing processes adopt individual control charts to monitor the process mean.

Tukey's control chart uses a single observation to monitor the process mean, thus making it suitable for monitoring destructive testing processes. Tukey's control chart utilizes quartiles to set up control limits, making it easier to use and distinguishing it from CUSUM and EWMA control charts. Alemi [1] applied Tukey's control chart to the health-care industry to monitor patient health, but at that time, the design method of Tukey's control chart had not yet been investigated. Thus, Alemi [1] is not necessarily applicable to industrial processes. Consequently, Tukey's control chart must be re-designed according to respective process characteristics in order to detect process variation effectively.

At the outset, control chart design must decide control limit width, sampling interval and sample size. Duncan [7] developed a cost model aimed at $\overline{X}$ chart and its optimal design from an economic perspective. Duncan's cost model components include process variation cost, false alarm cost, assignable cause search cost, sampling, inspection and plotting cost [7]. Through optimization technique, minimized total cost is found results in an optimal control chart design. Subsequent research has used Duncan's concept [7] to construct cost models aimed at different control charts and to determine optimal design [2, 3, 6, 10, 11 12]. Briefly, Tukey's control chart is an individual control chart which distinctively need not consider the decision of sample size in the design of control chart; and only requires the solution of control limit width and sampling interval.

Another unique idiosyncrasy of Tukey's control chart is that its construction of cost models must apply the error probabilities of the control charts to calculate the cost components. However, there have been no studies of calculations of error probabilities using Tukey's control chart to date. This reveals that the calculations of error probabilities of Tukey's control chart must first be built to fully construct the cost model.

* Corresponding author: g9321801@yuntech.edu.tw

In this study, the cost function of Tukey's control chart is constructed using Duncan's [7] concept, and the optimal design of Tukey's control chart is determined by economic design approach. Normal distribution is usually an important assumption of a control chart. We construct the error probabilities of Tukey's control chart under normal conditions, and apply them to the cost function. The wirebonding process of IC packaging uses the destructive testing approach to measure the ball shear strength. The monitoring of ball shear strength serves as an example of the application of an economic design of Tukey's control chart. Sensitivity analyses have been constructed to evaluate the effects of model parameters on the optimal design of Tukey's control chart.

2. TUKEY'S CONTROL CHART

Tukey's control chart is an individual control chart that applies the principle of Box plot to set up its control limits. The setup of Tukey's chart is as follows:

Step 1. Calculate the first quartile Q_1 and the third quartile Q_3.
Step 2. Calculate Inter-Quartile Range (IQR; $IQR = Q_3 - Q_1$).
Step 3. Use the following equation to construct upper-control limit (UCL) and lower-control limit (LCL).

$$\begin{aligned} \mathrm{LCL} &= Q_1 - k \times IQR \\ \mathrm{UCL} &= Q_3 + k \times IQR \end{aligned} \tag{1}$$

where parameter k determines the width of control limits and its default is 1.5 [1]. A region of a control chart between UCL and LCL is called the in-control region. If an observation falls within this region, the process is determined to be an in-control process. If an observation falls outside the control limits, the process is determined to be a mean-shift occurrence.

3. ECONOMIC MODEL CONSTRUCTION

3.1 Type I and type II error probabilities

Many studies consider normal distribution to always be a basic assumption of process observations [2, 4, 5, 8]. In a control chart, x is used to represent observations from a normal distribution with parameter μ and σ^2, and the initial process mean and variance are, respectively, μ_0 and σ^2. When process mean had shifted, the new process mean is $\mu_1 = \mu_0 + \delta\sigma$; here δ is the shift size ($\delta = (\mu_1 - \mu_0)/\sigma$). Let $P(\delta)$ be used to represent the probability that the observation falls outside the control limits when the shift size is δ. Using a normalization approach transfers x to z, $z = (x - \mu)/\sigma$; and the z follows standard normal distribution. The $P(\delta)$ is:

$$\begin{aligned} P(\delta) &= 1 - P\left(LCL \le x \le UCL \middle| \mu = \mu_1 = \mu_0 + \delta\sigma\right) \\ &= 1 - P\left(LCL - \delta\sigma \le x \le UCL - \delta\sigma \middle| \mu = \mu_0\right) \\ &= 1 - \{\Phi[(UCL - \mu_0)/\sigma - \delta] \\ &\qquad - \Phi[(LCL - \mu_0)/\sigma - \delta]\} \end{aligned} \tag{2}$$

where the Φ is a cumulative distribution function of a standard normal distribution [9]. On one hand, if δ=0, the $P(\delta=0)$ is the probability of type I error or false alarm rate. On the other hand, when process mean shifts $\delta\sigma$ units, the $P(\delta\neq0)$ is power. 1-$P(\delta\neq\delta)$ is the probability that the control chart does not detect the mean shift, and it is usually denoted as a type II error probability.

When the historical data is sufficiently high to fit the population probability distribution and can estimate its parameters, the quartiles can be calculated through the known probability distribution. Let x be the process observation following normal distribution with μ and σ, and use normalization approach transfers x to z. The inverse of a cumulative distribution function of a standard normal distribution is:

$$z = \Phi^{-1}(p) \tag{3}$$

where p is a specific probability. Let p be equal to 0.25 and 0.75 respectively. Use of Eq. (3) can calculate the first quartile Q_1 and the third quartile Q_3 of standard normal distribution. The Q_1 and Q_3 are equal to -0.67449 and 0.67449, respectively, and IQR = 1.34898. The control limits of Tukey's control chart, which is based on a standard normal distribution assumption, are:

$$\begin{aligned} \mathrm{UCL} &= 0.67449 + 1.34898k \\ \mathrm{LCL} &= -0.67449 - 1.34898k \end{aligned} \tag{4}$$

Using Eq. (2) and (4) can obtain the $P(\delta)$ of Tukey's control chart, as follows:

$$\begin{aligned} P(\delta) = 1 - [&\Phi(0.67449 + 1.34898k - \delta) \\ &- \Phi(-0.67449 - 1.34898k - \delta)] \end{aligned} \tag{5}$$

When $\delta = 0$, Eq.(5) becomes the type I error probability of Tukey's control chart, and its type II error probability is $1 - P(\delta \neq 0)$.

An example illustrating the application of Eq.(5) uses Tukey's control chart of k=1.5 to monitor the process mean and calculate its type I error probability. Let k and δ of Eq.(5) be respectively equal to 0 and 1.5, then the $P(\delta = 0)$ is approximately 0.00698.

3.2 Cost function

Duncan [7] presented a cost model for $\overline{X}$ chart that was much more realistic than other models. Duncan's model made some important assumptions: (1) A control chart only detects a single assignable cause at a time; (2) The time from in-control processes to out-of-control processes follows exponential distribution with parameter λ; (3) The control rule is only to be considered when an sample point falls outside the control limits; (4) The process allows continuous manufacturing operation while it is in the process of finding an assignable cause.

Furthermore, Duncan [7] divided the cycle time of process control into four periods:

(1) In-control period. The average time of in-control processes is approximately $1/\lambda$.

(2) The period of process variation detection. When assignable cause occurs, the probability using Tukey's control chart to successfully detect process mean shift is $P(\delta \neq 0)$. If the process mean shift occurs between the jth to j+1th sampling, and if the sampling interval is h, then the expected time between the jth to j+1th samplings for the process shift to occur is:

$$\tau = \frac{\int_{jh}^{(j+1)h} \lambda (t - jh) e^{-\lambda t} dt}{\int_{jh}^{(j+1)h} \lambda e^{-jt} dt} = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})} \quad (6)$$

Therefore, the time required for Tukey's control chart to detect the process mean shift is: $h/P(\delta \neq 0) - \tau$.

(3) The period of searching for an assignable cause and the period of restoring processes to an in-control state. The time of two period can be represented a constant D.

The expected length of a cycle time $E(T)$ is:

$$E(T) = 1/\lambda + h/P(\delta \neq 0) - \tau + D \quad (7)$$

The components of Duncan's cost model include: (1) the cost of an out-of-control state; (2) the cost of false alarm; (3) the cost of searching for an assignable cause; and (4) the sampling, inspection and plotting of cost. We modify these cost components based on the principle of Tukey's control chart as follows:

(1) Out-of-control cost. Once process variation occurs, the operation cost per hour is a_4. The manufacturing operation time consumed between onset of process variation occurrence to detection of variation is $E(T)-(1/\lambda)$. Thus, the total out-of-control cost is $a_4[E(T)-(1/\lambda)]$.

(2) The cost of false alarm. The cost of searching a false alarm is a_3. When an in-control process follows exponential distribution, the average number of samplings before mean shift is:

$$\int_{jh}^{(j+1)h} j\lambda e^{-\lambda t} dt = e^{-\lambda h} / (1-e^{-\lambda h}) \quad (8)$$

Since type I error probability of Tukey's control chart is $P(\delta = 0)$, the average occurrence number of false alarm is $P(\delta = 0)e^{-\lambda h}/(1-e^{-\lambda h})$, and the expected cost of false alarm is $a_3 P(\delta = 0)e^{-\lambda h}/(1-e^{-\lambda h})$.

(3) The cost to search an assignable cause and repair per time is set as a2 here.

(4) Since Tukey's control chart applies single observation to monitoring the process, the sampling, inspection and plotting cost is a constant a1. The sampling times during a cycle consist of $E(T)/h$; and thus the sampling, inspection and evaluation and plotting cost are $a_1 E(T)/h$.

The total cost is:

$$\begin{aligned} E(TC) = {} & a_1 E(T)/h \\ & + a_4[E(T)-(1/\lambda)] + a_2 \\ & + a_3 P(\delta = 0)e^{-\lambda h}/(1-e^{-\lambda h}) \end{aligned} \quad (9)$$

The expected cost per hour, denoted by $E(C)$, incurred by the process is:

$$E(C) = \frac{E(TC)}{E(T)} \quad (10)$$

where $P(\delta)$ and $P(\delta = 0)$ can be calculated from Eq.(5).

The economic design of Tukey's control chart is used to determine the appropriate values of h and k so that $E(C)$ may be minimized. In this study, the Solver Toolbox of Microsoft Excel 2003 is used as the model-solving tool. We have used this tool to solve the economic design model of Shewhart's $\overline{X}$ control chart as presented by Montgomery [9] [Example 9-5, p.464], and obtained the same results. Therefore, the Solver Toolbox of Microsoft Excel 2003 is suitable for use to solve the economic design model of a control chart.

4. A NUMERICAL EXAMPLE

4.1 Design of Tukey's control chart

Wirebonding is a key process in IC packaging and is the most common method for electrically connecting the aluminum bonding pads on a microchip surface to the package inner lead terminals on the lead-frame. Thermosonic ball bonding technology is applied to the wirebonding process. The thermosonic ball bonding uses a capillary tip made of tungsten carbide or ceramic material which feeds a fine diameter Au wire vertically through a hole in its center. The protruding wire is heated by a small flame or capacitor discharge spark, causing the wire to melt and form a ball at the tip. During bonding, the ultrasonic energy and the pressure cause a metallurgical bond to form between the Au wire and the Al pad. Upon completion of the ball bond, the bonding mechanism moves to the substrate inner lead pad and forms a thermocompression wedge bond. Up to this phase, the wire is broken and the tool continues to the next die bonding pad.

During the wirebonding process, the gold ball shear strength is an important quality characteristic and must be monitored effectively to stabilize IC quality. The destructive testing approach is utilized to measure the ball shear strength. Since the ball shear strength variance of the same IC is very small, previous sampling approaches sampled one IC and randomly selected one ball from it to perform testing. As a consequence, only single shear strength can be obtained during each testing and Tukey's control chart is selected for the monitoring.

Since historical observations obtains 100 testing values of ball shear under in-control process, these testing values are verified following normal distribution with $\mu = 18.6496$ (g) and $\sigma = 1.75416$ through application of the Kolmogoroc-Smirnov test (P-value= 0.377). Since experience can predict that the ball shear strength mean shift will occur in each 20 hours ($\lambda = 0.05$) with a shift size of about 2 ($\delta = 2$), the time from searching an assignable cause to the repair process to normal state is about 1 hour (D=1). The sampling, inspection and plotting cost is one dollar (a_1=1). The cost of searching a mean shift cause and of the repairing process is \$25 (a_2=25) each instance. Furthermore, during the control process, the control chart may show a false alarm due to sampling error, and the cost of a false alarm occurrence is \$50 (a_3=50). When the mean of gold ball shear strength has shifted but has not been detected, the cost of loss per hour caused by the continuous operation is \$100 (a_4=100).

Once the above parameters have been applied to Eq.(10), the Solver Toolbox of Microsoft Excel 2003 is used to find a solution for a cost-minimizing. The obtained optimal Tukey's control chart design is h=0.4653, k=1.2278; and $E(C)$ is 14.38, $P(\delta = 0) = 0.0198$, $P(\delta = 2) = 0.3707$. This means that sampling will be performed each 0.4653 hours, and the control limit width of Tukey's control chart should be set at 1.2278, as the probability of false alarm is 0.0198. Reasonably, when the process mean shifts by 2 standard deviations, the probability of detecting process mean shift is 0.3707, and accordingly, the expected cost per hour is \$14.38.

4.2 Monitoring the wirebonding process

Once a Tukey's control chart has been designed, its control limits and sampling interval can be set. The k=1.2278 and Eq.(4) can calculate UCL=2.33077 and LCL=-2.33077; and the sampling interval is determined at 0.4653 hours (about 30 minutes). We sample and test the gold ball shear strength per 30 minutes, using normalizing approach to transform the observation value to the z value and to plot the z value on Tukey's control chart as in Figure 1.

Figure 1 shows the monitoring of wirebonding processes during sampling 51 times. Point 34 falls outside control limits at the 17th hour. Since determining the cause indicates that the false alarm occurred during this time sampling, the process continues operation. Furthermore, point 51 also falls outside control limits at the 25.5th hour. We check immediately the operation procedure of this process and find the mean shift occurrence due to the machine sets error parameters. Once the parameters of the machine are adjusted to normal values, the process is repaired and returns to normal state.

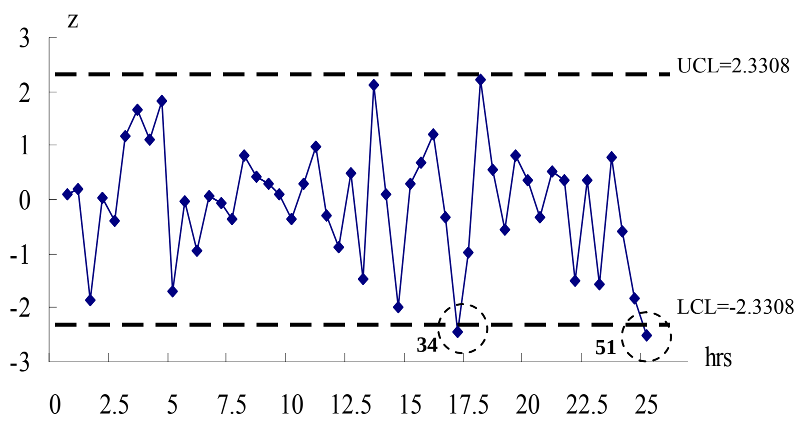

Figure 1. Tukey's control chart for a wirebonding process

4.3 Sensitivity analysis

In this section, the effects of model parameters on the optimal design of the Tukey's control chart will be investigated with the result listed in Table 1.

Table 1. Effects of model parameters on the optimal design of the Tukey's control chart

		h	k	Type I error	Power	$E(C)$
δ	1	0.4577	0.9061	0.0579	0.1868	22.52
	1.5	0.4302	1.0987	0.0310	0.2559	17.60
	2	0.4653	1.2278	0.0198	0.3707	14.38
	3	0.5668	1.4398	0.0089	0.6493	11.01
λ	0.01	0.9476	1.2467	0.0185	0.3608	5.29
	0.05	0.4653	1.2278	0.0198	0.3707	14.38
	0.1	0.3615	1.2075	0.0213	0.3808	22.38
	0.5	0.2866	1.0813	0.0329	0.4471	56.56
D	0.5	0.4521	1.2335	0.0194	0.3675	12.34
	1	0.4653	1.2278	0.0198	0.3707	14.38
	2	0.4917	1.2152	0.0207	0.3769	18.19
	10	0.7106	1.1330	0.0276	0.4197	39.61
a_1	0.1	0.0872	1.7664	0.0022	0.1452	10.71
	1	0.4653	1.2278	0.0198	0.3707	14.38
	10	2.2733	0.5479	0.1575	0.7215	21.50
a_2	2.5	0.4625	1.2273	0.0198	0.3707	13.36
	25	0.4653	1.2272	0.0198	0.3707	14.38
	250	0.4967	1.2261	0.0199	0.3713	24.58
a_3	5	0.6584	0.5806	0.1449	0.7065	11.11
	50	0.4653	1.2278	0.0198	0.3707	14.38
	500	0.3017	1.7519	0.0024	0.1497	20.37
a_4	10	1.8916	1.1812	0.0233	0.3944	3.96
	100	0.4653	1.2278	0.0198	0.3707	14.38
	1000	0.1389	1.2390	0.0190	0.3647	76.30

From table 1, it can be seen that when large δ occurs, wider control limits and lower cost will be obtained, and the false alarm rate will be lower. If the λ increases (i.e., the expected time of in-control state decreases), shorter sampling interval and higher cost will be obtained. When D increases, longer sampling interval and higher cost will be obtained. Simultaneously, if larger a_1 occurs, longer sampling interval and narrower control limits will be obtained. Furthermore, change of a_2 does not have too much effect on the error probabilities and sampling interval, but the cost will increase following the increase of a_2. If a_3 increases, shorter sampling interval and wider control limits will be obtained. When larger a_4 occurs, shorter sampling interval will be obtained, and the cost conspicuously increases.

5. Conclusion

To be brief, in this study, the economic design model of Tukey's control chart is strongly based on Duncan's [7] cost model concept. The calculations of error probabilities are simply constructed and applied to the economic design model. The economic design of Tukey's control chart is applied to a real-world case of the wirebonding process of IC packaging; and obtains the optimal design of Tukey's control chart to monitor the gold ball shear strength.

Based on the results of sensitivity analyses some important conclusions of Tukey's chart design may be drawn as follows:

(1) When large δ is used, wider control limits and lower cost will be obtained.

(2) If the time of in-control process decreases, shorter sampling interval and higher cost will result.

(3) When a larger D value is used, longer sampling interval and higher cost will result.

(4) If larger a_1 occurs, longer sampling interval and narrower control limits will result.

(5) The change of a_2 has insignificant effect on the design of Tukey's control chart.

(6) If larger a_3 is used, shorter sampling interval and wider control limits will result.

(7) When a_4 increases, shorter sampling interval and higher cost will result.

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ABOUT THE AUTHORS

Chau-Chen Torng is an associate professor in the Graduate School of Industrial Engineering and Management at National Yunlin University of Science and Technology, Taiwan. He received his M.S. degree in Department of Industrial and Engineering from Auburn University and Ph.D. degree in Department of Industrial Management and Systems Engineering from the Arizona State University. His present research interests include six sigma, statistical process control and Response Surface Methodology.

Pei-Hsi Lee is a Ph.D. student in the Graduate School of Industrial Engineering and Management at National Yunlin University of Science and Technology, Taiwan. He received his M.S. degree in Graduate School of Business Administration from Shu-Te University. His research activities include statistical process control and reliability engineering.

Chun-Chieh Tseng is interested in Quality Engineering and Total Quality Management.

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Tukey管制圖的經濟性設計

童超塵、李佩熹*、曾君爵
國立雲林科技大學工業工程與管理所
640雲林縣斗六市大學路三段123號

摘要

Tukey管制圖包含了數種特性：容易設置、使用個別值觀測監控製程、適合監控破壞性檢驗製程。在運用Tukey管制圖監控製程之前，管制界限寬度、抽樣間隔必須要決定。在這研究中，Duncan的成本函數被應用來建構Tukey管制圖的經濟性設計模式，以獲得最佳管制圖設計，此外，本研究在常態條件假設之下，建構Tukey管制圖的誤差機率計算方法，並將此誤差機率的計算應用到Tukey管制圖的經濟性設計模式中。最後，應用一個IC封裝的真實案例說明Tukey管制圖的經濟性設計模式的使用，從這案例的敏感度分析結果發現到，當管制內時間太短，或者的製程變異成本太大，都會明顯增加製程管制成本。

關鍵詞：Tukey管制圖，經濟性設計，四分位距，盒形圖
(*聯絡人: g9321801@yuntech.edu.tw)