Research Article

Quality and Reliability Engineering International

(wileyonlinelibrary.com) DOI: 10.1002/qre.1664

Published online in Wiley Online Library

Performance of Tukey's and Individual/Moving Range Control Charts

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This paper compares two control charts: Tukey (TCC) and individual/moving range (XmR) control charts. Both are designed to examine single observation per time period, but little is known about which one is more efficient and under what conditions. We simulated data from different distributions and examined the performance of the two control charts on these data. Performance was assessed using the of average run length, extra quadratic loss, median run length, standard deviation run length, performance comparison index, and relative average run length. Overall, TCC was more efficient than XmR, when observations had binomial, Rayleigh, logistic, lognormal, Maxwell, normal, Poisson, Weibull (with $\alpha = 10$, $\beta = 1$), and Student's *t* (30 and 10 degrees of freedom) distributions. XmR was more efficient when observations had Student's *t* (with 4 degrees of freedom) and gamma (with $\alpha = 4$, $\beta = 1$) distributions. These results suggest that improvement teams could reach faster conclusions if they use TCC in most common situations. Copyright © 2014 John Wiley & Sons, Ltd.

Keywords: average run length; individual and moving range control chart; Tukey control chart; extra quadratic loss; median run length; standard deviation run length

1. Introduction

In many processes, there is one observation per time period. In these situations, the process is typically analyzed using the individual/moving range (XmR) control chart. In recent years, Tukey control chart (TCC) has been proposed for the analysis of the same type of data. The purpose of this paper is to compare these two methods and identify the environment where one method may be preferred to the other.

The XmR chart has been used since 1942 (Dudding and Jennett,[1] Keen and Page,[2] Crowder,[3,4] Gitlow *et al.*,[5] Nelson,[6,7] and Wheeler[8]). The performance of the XmR chart has been examined by several authors (Roes *et al.*[9], Rigdon *et al.*,[10] Acosta-Mejıa,[11] Sargut and Demirors,[12] Khoo *et al.*,[13] Trip and Wieringa,[14] Borneman,[15] Chakraborti,[16] Benneyan,[17] Marks and Krehbiel,[18] and Poots and Woodcock[19]). To date, no comparison has been made to TCC. TCC was proposed by Alemi[20] and based on Tukey[21] fences for box plots. Borckardt *et al.*,[22] Borckardt *et al.*,[23] Torng and Lee,[24] Torng *et al.*,[25] and Tercero-Gomez *et al.*[26] examined the performance of TCC. Lee[27] introduced asymmetrical control limits to monitoring the process and showed that it is less sensitive to signal mean shifts, when the monitoring variable follows a skewed distribution. Sukparungsee[28] also reports that TCC is robust in detecting changes in parameters of skew distributions. Its performance was superior to the exponentially weighted moving average control chart. Tercero-Gomez *et al.*[26] introduced a method of handling skewed distribution in TCC. Lee and Torng,[29] Lee *et al.*,[30] and Sukparungsee[31] further introduced some modification of the Tukey chart to improve its performance. These studies while examining and improving the performance of TCC have not compared it to the XmR chart.

2. Illustrative example using real life data

To illustrate the use of these two control charts, we present the analysis of data taken from the Major League Baseball to detect whether use of steroids has led to changes in scores. Major League is the highest professional baseball league in the USA. It has experienced a series of historical eras since it begun. Using data from 1969 to 2008, Hill and Schvaneveldt[32] proposed that the steroid era in Major League can be detected by comparing scores in this era to the historical pattern. They used an XmR chart and assumed that 1968 to 1992 could be used as baseline data, when no steroids were used. Figure 1 shows the performance of TCC and XmR

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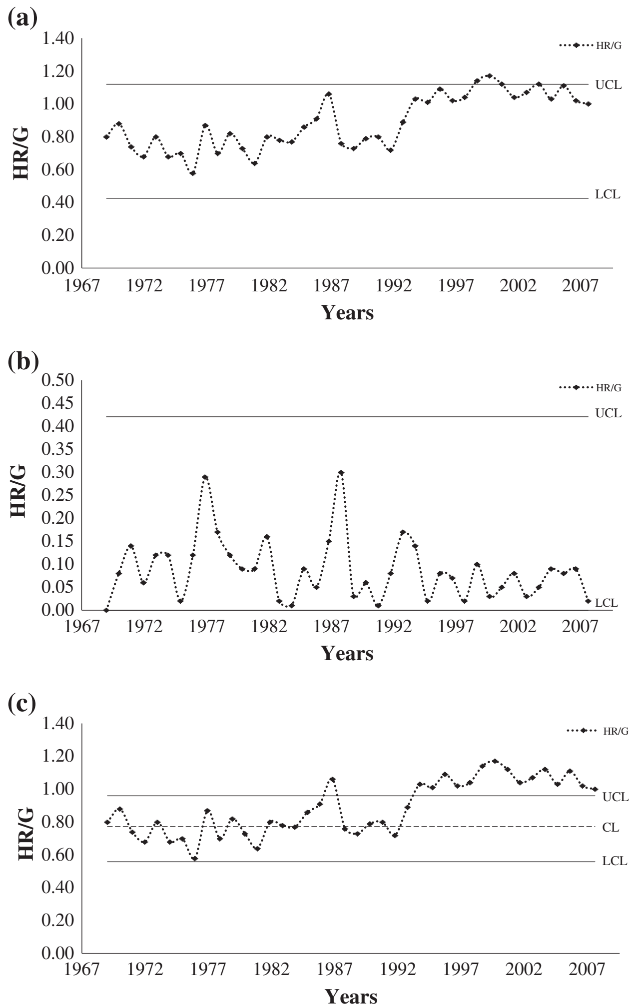

Figure 1. Control charts of (a) individual, (b) moving range Xmr, and (c) Tukey (TCC) charts using real life data

charts on these data. In this example, TCC detects out of control process sooner than XmR. It signals an out of control process in the 34th observation. While this example shows that TCC is more efficient, other data can be organized where XmR is more efficient. The real question is when one approach is preferred to another.

3. Methods

Using different distributions, we simulated data and then tested if the data were in or out of control by comparing the observations to the control limits. The limits of TCC were based on the following formulas:

$$\left.\begin{aligned} LCL_{TCC} &= Q_1 - K_{TCC}(IQR) \\ CL_{TCC} &= Q_2 \\ UCL_{TCC} &= Q_3 + K_{TCC}(IQR) \end{aligned}\right\} \quad (1)$$

where Q_2 = median, Q_1 = first quartile, Q_3 = third quartile, and IQR = interquartile range. K_{TCC} determines the width of control limits. Alemi[20] used K_{TCC} = 1.5, but it may be readjusted based on type-1 error rate.

For XmR, there are really two control charts. The plotting statistic for individual observations is the observation x, and the plotting statistic for moving range is the range between consecutive values. Control limits for combined individual and XmR charts can be calculated as follows:

Control limits for individual x chart

$$\left.\begin{aligned} UCL_X &= \mu_0 + M_0\sigma_0 \\ LCL_X &= \mu_0 - M_0\sigma_0 \end{aligned}\right\} \quad (2)$$

Table I. (a) ARL for I/MR and TCC using exponential, Laplace, logistic, lognormal, normal, and Rayleigh distribution.

Distribution	Exponential (1)		Laplace (0, 1)		Logistic (6, 2)		Lognormal (0, .25)		Normal (0, 1)		Rayleigh (2)	
Δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
0	370.9496	369.5401	370.9715	370.0389	370.8961	369.9821	370.1656	369.9439	370.841	370.8165	370.1061	370.5704
0.25	283.3029	288.3783	351.8152	352.4339	349.3092	332.564	260.15	245.6132	347.3143	287.5373	320.3936	211.0232
0.5	219.2788	219.6093	309.7616	298.6532	273.7054	258.0169	180.3509	165.9901	290.8781	157.964	248.5413	126.8717
0.75	173.782	177.9641	235.9164	225.7889	202.4771	180.2914	122.3967	112.1288	206.9134	82.8161	174.0174	77.8845
1	132.7663	136.4251	174.6177	169.0388	138.2585	116.4622	82.5952	76.2319	129.8928	43.3162	112.4684	48.1536
1.25	103.7133	106.6893	124.9695	123.812	92.6711	76.5686	55.6101	52.4463	75.1629	24.9387	70.1537	30.9304
1.5	80.9012	82.5991	91.6928	87.6054	60.5578	49.0473	37.6212	36.7613	42.2642	14.7655	44.2499	20.4655
1.75	63.9425	64.6892	61.7022	62.3634	39.3755	31.912	26.6896	25.1655	24.6533	9.4826	28.0154	13.793
2	49.6916	50.5322	44.1403	42.7649	25.5403	20.721	18.4848	17.3641	14.9886	6.4049	18.7478	9.5748
2.25	38.5941	38.8331	31.7002	30.3701	16.6459	13.5767	12.9814	12.1786	9.4462	4.3814	12.8113	6.9543
2.5	29.7367	30.5485	22.1234	22.0303	10.8448	8.9692	9.3331	8.7581	6.2499	3.276	8.9834	5.045
2.75	23.5015	23.6405	15.5096	15.0749	7.3898	6.0543	6.7561	6.3091	4.4581	2.4821	6.4935	3.8064
3	18.1349	18.4659	11.0799	10.7631	4.9629	4.308	4.9559	4.705	3.2436	2.0214	4.7307	2.9811
3.25	14.1193	14.7439	7.6408	7.5017	3.5461	3.0597	3.7567	3.5133	2.4983	1.6864	3.6071	2.3517
3.5	11.0465	11.201	5.3736	5.2248	2.5752	2.2617	2.8302	2.709	2.0292	1.4536	2.8209	1.9306
3.75	8.5705	8.6312	3.808	3.6424	2.0374	1.8022	2.2617	2.1309	1.6768	1.2939	2.2842	1.6151
4	6.6069	6.8587	2.6722	2.5802	1.6659	1.5381	1.8153	1.7497	1.4406	1.1877	1.8614	1.396

Table I. (b) ARL for I/MR and Tukey control chart using binomial, geometric, Poisson, and Student's *t* distributions

Distribution	Binomial (50, .3)		Geometric (.3)		Poisson (10)		*t* (30)		*t* (10)		*t* (4)	
Δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
0	370.4029	370.1301	370.5545	370.958	370.2667	370.0372	370.4293	369.7871	370.7405	370.0975	369.7622	370.9171
0.25	284.3578	235.1747	293.9788	286.732	234.5966	208.0929	334.7008	298.2672	341.1457	337.3506	350.4241	357.9335
0.5	188.1636	128.6442	217.2202	230.6967	130.3063	115.6001	273.2622	188.5556	287.8007	251.6146	298.9259	330.1111
0.75	112.0443	69.9143	174.2648	179.0771	69.4064	68.4419	188.1562	107.348	222.5579	172.4296	221.8205	303.8179
1	63.7108	38.8802	141.4198	141.0443	50.742	41.6395	117.9002	59.8782	157.3087	110.5829	172.2773	258.1863
1.25	36.9385	23.2413	103.7619	108.5833	36.4914	25.6141	71.1755	35.0302	107.9931	69.8196	125.0862	211.5859
1.5	21.991	14.3107	80.336	84.005	20.9325	16.7384	41.5391	21.219	69.2231	43.2026	88.3826	167.5096
1.75	13.7623	9.3474	66.8682	64.4547	12.0344	10.9729	25.1708	13.0205	43.5085	27.1575	61.772	131.5081
2	8.8858	6.4106	50.3702	50.8845	7.4948	7.536	15.2719	8.3674	27.6491	17.3488	44.0425	100.8229
2.25	6.1762	4.4063	37.5636	40.522	6.4435	5.3359	9.774	5.6778	17.6189	11.2082	30.7168	74.6554
2.5	4.2897	3.3507	31.4575	30.4476	4.696	3.8698	6.5949	4.0069	11.495	7.3156	21.3312	53.4161
2.75	3.1958	2.5402	24.4101	24.9258	3.2796	2.9808	4.5606	2.9571	7.5272	5.0176	15.273	37.6994
3	2.5071	2.0496	18.0682	18.7186	2.3937	2.3776	3.3156	2.3101	5.1441	3.5683	10.6595	25.6055
3.25	2.0013	1.7025	14.5651	14.7071	1.8874	1.8905	2.5334	1.8562	3.6379	2.7034	7.6334	17.4613
3.5	1.6964	1.4814	11.8613	11.3881	1.7721	1.5951	1.9903	1.5851	2.6927	2.0697	5.2562	11.8802
3.75	1.4552	1.3067	8.6766	8.8162	1.4909	1.3893	1.6594	1.3779	2.1093	1.6848	3.7364	7.8118
4	1.2961	1.1944	6.6029	6.916	1.2736	1.2375	1.432	1.2464	1.7116	1.4538	2.5726	5.1294

Table I. (c) ARL for I/MR and Tukey control chart using gamma and Weibull distributions

Distribution	Gamma (4, 1)		Gamma (2, 1)		Gamma (1, 1)		Weibull (10, 1)		Weibull (3.5, 1)		Weibull(2, 1)	
Δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
0	369.8545	370.4001	370.3149	370.4443	370.0794	370.8007	370.4482	370.4692	369.9316	370.5405	370.2268	370.1607
0.25	250.26	292.2642	272.189	270.6067	283.9058	288.8744	586.4621	229.3811	202.0015	194.3793	211.7797	215.3696
0.5	176.9297	203.3939	199.5599	197.8237	223.7072	223.2161	904.1804	85.8268	94.0578	91.5664	126.3051	128.2205
0.75	118.5689	137.0673	145.3865	146.0138	171.9289	176.4225	1195.187	32.9594	47.8126	45.8224	78.1694	76.2828
1	82.6578	93.9178	106.8749	109.0542	134.8576	138.9957	1087.114	15.0858	25.6474	24.5759	48.5641	49.2572
1.25	57.704	66.466	78.0593	76.8737	104.4008	106.3598	510.8429	8.0426	15.0818	14.3891	31.393	31.2328
1.5	41.0552	46.4198	58.3683	57.8209	79.9236	83.0901	159.9641	4.927	9.1493	9.1152	20.4162	20.6124
1.75	28.8011	33.0215	43.1852	42.8321	62.9344	63.9344	55.9573	3.3678	6.1837	5.9754	13.8872	13.8619
2	20.6279	23.1301	31.6286	32.17	49.2744	49.727	22.8769	2.4698	4.3791	4.2724	9.7013	9.4496
2.25	14.7888	16.7834	23.9705	23.5463	37.9982	38.6647	11.1409	1.9807	3.166	3.1444	6.9917	6.8894
2.5	10.6407	12.0481	17.6482	17.6389	29.6345	30.4112	6.4053	1.6449	2.4508	2.4402	5.1402	5.0472
2.75	7.8837	9.0215	13.3285	13.3072	23.4081	23.0358	4.1081	1.4463	2.0147	1.9943	3.8267	3.8699
3	5.8826	6.6001	9.9448	10.0228	18.2302	18.6415	2.9146	1.3037	1.6865	1.6842	2.9458	2.988
3.25	4.4466	4.9151	7.5468	7.5563	14.1749	14.4274	2.204	1.2157	1.4539	1.4507	2.3644	2.3692
3.5	3.4059	3.7279	5.7572	5.7234	11.0252	11.1977	1.8044	1.1414	1.3064	1.3143	1.9054	1.9495
3.75	2.6151	2.8978	4.4281	4.4087	8.523	8.5988	1.5438	1.0957	1.1952	1.1913	1.6377	1.6363
4	2.0786	2.3133	3.3623	3.4381	6.5842	6.7332	1.3821	1.0683	1.1174	1.1234	1.4001	1.3948

Table I. (d) ARL for I/MR and Tukey control chart using chi-square and Maxwell distributions

Distribution	Chi-square (100)		Chi-square (10)		Maxwell (2)	
Δ	Xmr	TCC	Xmr	TCC	Xmr	TCC
0	370.679	370.3389	369.7236	370.3621	370.4431	370.066
0.25	211.9161	209.7514	252.3117	255.0451	167.4929	79.2032
0.5	119.4787	120.8631	169.0671	168.8236	43.358	20.7911
0.75	69.9489	69.1689	115.4225	113.8815	12.7974	6.9714
1	41.3117	41.8852	77.6702	78.0585	4.7978	3.0344
1.25	26.4515	26.2358	53.3816	54.2645	2.3139	1.6722
1.5	16.7663	16.9257	36.6018	36.7234	1.4128	1.1688
1.75	10.8771	10.7916	25.7153	26.2419	1.082	1.0208
2	7.4295	7.4716	18.1449	18.3592	1.0034	1
2.25	5.2562	5.2626	12.9918	13.0733	1	1
2.5	3.9496	3.8538	9.4287	9.4118	1	1
2.75	2.9462	2.954	6.8959	7.0315	1	1
3	2.3044	2.2737	5.1878	5.0641	1	1
3.25	1.8706	1.8432	3.8861	3.9251	1	1
3.5	1.5791	1.5569	3.0284	3.0413	1	1
3.75	1.3751	1.3778	2.3646	2.3534	1	1
4	1.2413	1.2268	1.9077	1.9046	1	1

Control limits for moving range

$$\left.\begin{array}{l} UCL_R = R_0\sigma_0 \\ LCL_R = 0 \end{array}\right\} \quad (3)$$

The performance of control charts was assessed using the concept of average run length (ARL), extra quadratic loss (EQL), median run length (MDRL), standard deviation run length (SDRL), performance comparison index (PCI), and relative ARL (RARL). These measures of efficiency are described in the succeeding texts:

3.1. Average run length

Average run length$_0$ is the average number of samples until an out of control shift is detected by the chart when the process mean is in control. ARL_1 is the average number of samples until an out of control signal is detected by the chart when the process mean is moved to an out of control situation. For calculating ARL of combined the individual and moving range chart, we used the procedure introduced by Crowder.[3,4] Marks and Krehbiel[18] also used these parameters for calculating ARL for XmR charts. ARL calculations for TCC were based on the method used by Torng and Lee.[24] For Weibull and Rayleigh distributions, we used asymmetrical control limits suggested by Lee.[27] We have also applied continuity correction for discrete distribution as suggested by Chan *et al.*[33]

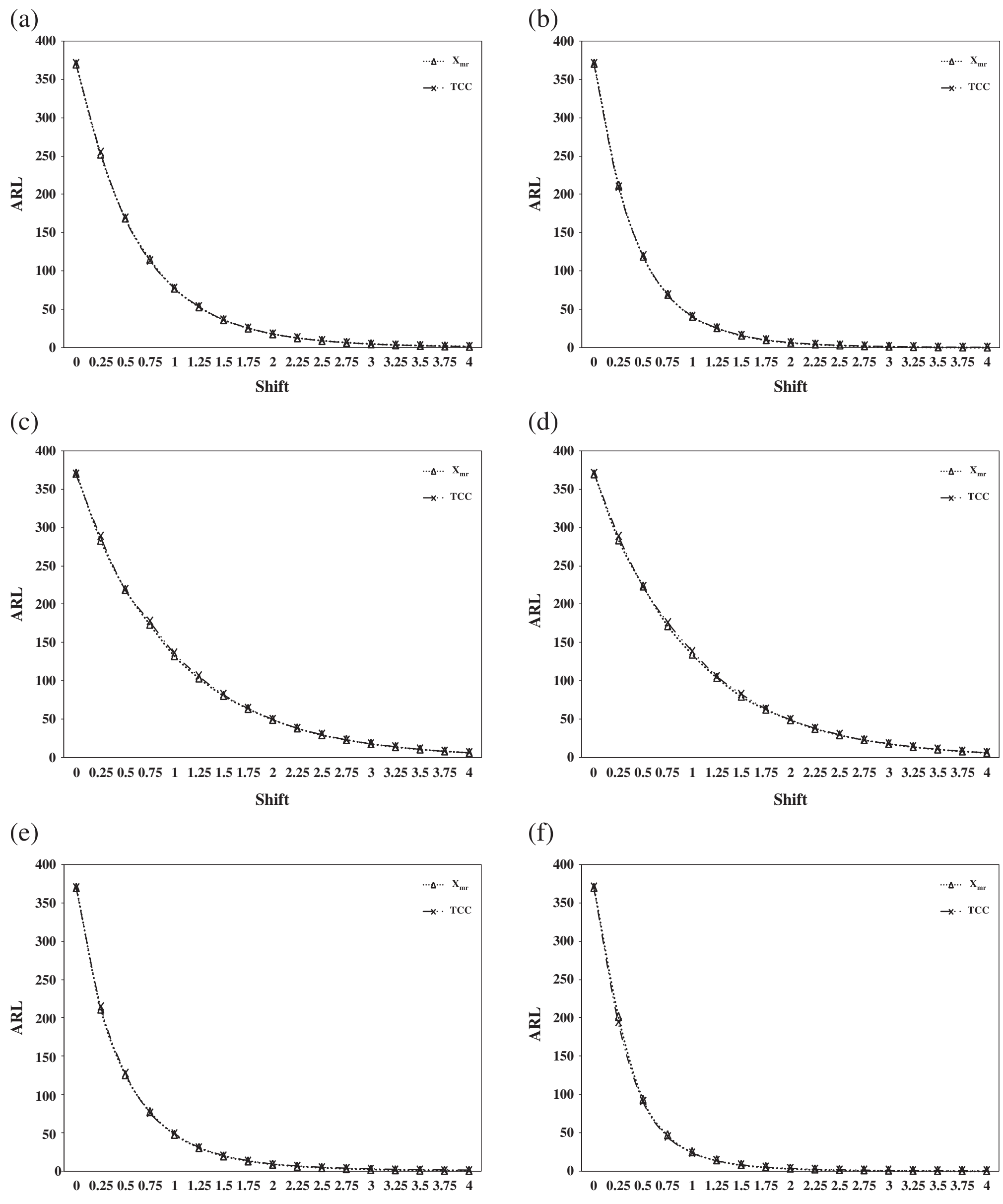

Figure 2. Both charts with almost the same efficiency: (a) chi-square (10), (b) chi-square (100), (c) exponential (1), (d) gamma (1, 1), (e) Weibull (2,1), and (f) Laplace distribution

Table II. (a) EQL, RARL, and PCI for I/MR and Tukey control chart using chi-square, exponential, Laplace, lognormal, normal, and Rayleigh distributions

Distribution	Exponential (1)		Laplace (0, 1)		Logistic (6, 2)		Lognormal (0, .25)		Normal (0, 1)		Rayleigh (2)		Chi-square (100)	
Δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
EQL	138.858	141.496	119.442	116.423	75.612	63.8	55.027	51.753	57.803	26.072	62.177	32.282	26.784	26.712
RARL	1	1.018	1.026	1	1.163	1	1.061	1	2.007	1	1.81	1	1.004	1
PCI	1	1.019	1.026	1	1.185	1	1.063	1	2.217	1	1.926	1	1.003	1

Table II. (b) EQL,RARL, and PCI for I/MR and Tukey control chart using binomial, chi-square, geometric, Poisson, and Student's *t* distributions

Distribution	Binomial (50, .3)		Geometric (.3)		Poisson (10)		*t* (30)		*t* (10)		*t* (4)		Chi-square (10)	
Δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
EQL	34.625	24.551	141.828	143.971	30.568	26.723	56.049	32.949	82.696	57.011	116.984	234.732	54.245	54.518
RARL	1.346	1	1	1.015	1.124	1	1.58	1	1.393	1	1	1.934	1	1.004
PCI	1.41	1	1	1.015	1.144	1	1.701	1	1.451	1	1	2.007	1	1.005

Table II. (c) EQL,RARL, and PCI for I/MR and Tukey control chart using chi-square, gamma, and Weibull distributions

Distribution	Gamma (4, 1)		Gamma (2, 1)		Gamma (1, 1)		Weibull (10, 1)		Weibull (3.5, 1)		Weibull (2,1)		Maxwell (2)	
δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
EQL	59.907	67.926	89.311	89.208	138.376	140.755	229.686	12.939	18.01	17.647	32.449	32.423	7.511	6.389
RARL	1	1.129	1.001	1	1	1.016	16.489	1	1.018	1	0.999	1	1.267	1
PCI	1	1.134	1.001	1	1	1.017	17.752	1	1.021	1	1.001	1	1.176	1

3.2. Extra quadratic loss

Extra quadratic loss is the overall performance measure of a particular chart. EQL is defined as the weighted average of ARL over the entire shift domain $\delta_{\min} < \delta < \delta_{\max}$ using the δ^2 as a weight. EQL can be written in the form

$$EQL = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \delta^2 ARL(\delta) d\delta \Big\} \quad (4)$$

where $ARL(\delta)$ is the ARL of a particular chart at δ. Equation (4) is based on the assumption that δ has a uniform distribution over the interval $[\delta_{\max} - \delta_{\min}]$ further discussed by Ahmad *et al.* [34–36]

3.3. Median run length and standard deviation run length

Maravelakis *et al.*[37] suggested the MDRL and SDRL in the case of the skewed nature of a run length distribution. For better performance, low values of MDRL and SDRL are desirable.

3.4. Performance comparison index

This measure of efficiency is the ratio of EQL and EQL of the best chart under the similar situation. Ou *et al.*[38] discussed this measure in another form using a similar ratio. PCI supports the performance comparison and ranking-based EQL. Charts that have the lowest EQL have PCI = 1, and all other charts have a PCI greater than 1. A PCI formula has been further written in the succeeding texts.

$$PCI = \frac{EQL}{EQL_{Best.Chart}} \Big\} \quad (5)$$

3.5. Relative average run length

Relative average run length computes the average ratios between $ARL(\delta)$ to the $ARL_{bmk}(\delta)$. For a chart with the lowest $ARL_{bmk}(\delta)$, RARL is 1, and all other charts have an RARL greater than 1 (Ahmad *et al.*[34–36]).

$$RARL = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \frac{ARL(\delta)}{ARL_{bmk}(\delta)} d\delta \Big\} \quad (6)$$

To find out these performance measures, we generated the process data using different probability distributions: binomial, chi-square, exponential, gamma, geometric, Laplace, logistic, lognormal, Maxwell, normal, Poisson, Rayleigh, Student's *t*, and Weibull distributions.

4. Results and discussion

Tables I(a)–I(d) shows the ARL for XmR and TCC in data simulated from different distributions. These data are also presented in Figure 2 graphically. The performance of the two control charts is similar with some exceptions. The ARL performance of both control charts is similar using geometric, exponential, Laplace, Weibull (2, 1), gamma (2, 1), gamma (1, 1), and chi-square (100, 10 degrees of freedom) distributions. For comparison purposes, refer to Figure 2.

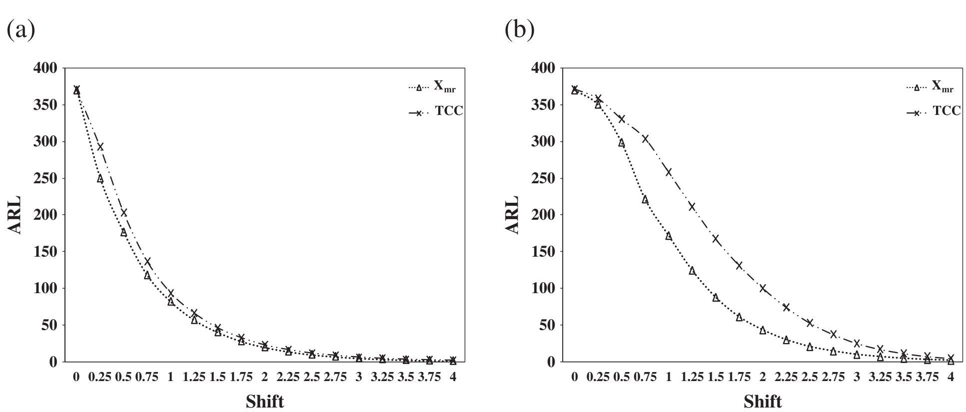

Figure 3. Individual/moving range (I/MR) chart with more efficiency than Tukey chart: (a) gamma (4, 1) and (b) *t* (4) distribution

Table III. (a) MDRL performance of Tukey and I/MR chart using binomial, geometric, Poisson, and Student's *t* distributions

Distribution	Binomial (50, .3)		Geometric (.3)		Poisson (10)		*t* (30)		*t* (10)		*t* (4)	
Δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
0	255	254	247.5	245	247	250	256.5	249	260	258	250	260
0.25	194	162	200	193	157	140	231	204	235	237	243	247
0.5	126	88	147	150	89	77	186	130	201	174	206	230
0.75	76	47	116	117	48	47	131	74	155	120	153	212
1	43	26	96	92	33	28	81	41	107	76	121	179
1.25	25	16	72	71	25	17	49	24	73	49	84	146
1.5	15	10	55	55	15	11	29	15	48	31	61	118
1.75	10	7	46	42	9	8	18	9	30	19	42	89
2	6	5	35	34	5	5	11	6	19	12	30	69
2.25	4	3	25	26	4	4	7	4	12	8	21	51
2.5	3	2	21	21	3	3	5	3	8	5	15	36.5
2.75	2	2	17	17	2	2	3	2	5	4	11	26
3	2	2	13	13	2	2	2	2	4	3	7	18
3.25	1	1	10	10	1	1	2	1	3	2	5	12
3.5	1	1	8	8	1	1	1	1	2	2	4	8
3.75	1	1	6	6	1	1	1	1	2	1	3	6
4	1	1	5	5	1	1	1	1	1	1	2	4

Table III. (b) MDRL performance of Tukey and I/MR chart using gamma and Weibull distributions

Distribution	Gamma (4, 1)		Gamma (2, 1)		Gamma (1, 1)		Weibull (10, 1)		Weibull (3.5, 1)		Weibull (2, 1)	
δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
0	260	252	260	251	258	198	260	256	257	254	254	258
0.25	175	198	175	188	199	196	397	159	137	133	148	150
0.5	122	142	122	137	154	153	613.5	59	68	63	88	89
0.75	81	95	81	101	118	124	824	23	33	32	54	53
1	57	65	57	76	92	97	741	11	19	17	34	35
1.25	40	45	40	52	73	74	346	6	11	10	22	21
1.5	28	32	28	40	55	58	111	3	7	7	14	14
1.75	20	23	20	29	43	45	39	2	4	4	10	10
2	14	16	14	23	34	34	16	2	3	3	7	7
2.25	10	12	10	16	26	27	8	1	2	2	5	5
2.5	8	9	8	12	21	21	5	1	2	2	4	4
2.75	6	6	6	9	16	16	3	1	2	2	3	3
3	4	5	4	7	13	13	2	1	1	1	2	2
3.25	3	4	3	5	10	10	2	1	1	1	2	2
3.5	3	3	3	4	8	8	1	1	1	1	1	1
3.75	2	2	2	3	6	6	1	1	1	1	1	1
4	2	2	2	3	5	5	1	1	1	1	1	1

Table III. (c) MDRL performance of Tukey and I/MR chart using exponential, Laplace, logistic, normal, and Rayleigh distributions

Distribution	Exponential (1)		Laplace (0, 1)		Logistic (6, 2)		Lognormal (0, .25)		Normal (0, 1)		Rayleigh (2)	
δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
0	261.5	251	259	256	255	253	256	257	257	256	259	256
0.25	201	205	244	239.5	246	228	179	169	237	197	225	149
0.5	150	150	218	205	188	178	123	113	197	110	171	89
0.75	120	123	164	154	141	125	85	77	143	56	118	53
1	92	95	124	115	95	80	57	53	89	31	79	33
1.25	72	72	86	86	64	53	39	36	52	17	49	21
1.5	56	57	64	59	42	34	26	26	29	10	31	14
1.75	44	45	43	43	28	22	18	17	17	7	20	10
2	35	35	31	30	18	14	13	12	11	5	13	7
2.25	27	27	22	21	12	9	9	8	7	3	9	5
2.5	21	21	16	16	8	6	7	6	5	2	6	4
2.75	16	16	11	11	5	4	5	5	3	2	5	3
3	13	13	8	8	4	3	4	3	2	2	3	2
3.25	10	10	5	5	3	2	3	3	2	1	3	2
3.5	8	8	4	4	2	2	2	2	2	1	2	1
3.75	6	6	3	3	2	1	2	2	1	1	2	1
4	5	5	2	2	1	1	1	1	1	1	1	1

Table III. (d) MDRL and SDRL performance of Tukey and I/MR chart using chi-square and Maxwell distributions

	MDRL performance using continues distribution						SDRL performance using continues distribution					
Distribution	Chi-square (100)		Chi-square (10)		Maxwell (2)		Chi-square (100)		Chi-square (10)		Maxwell (2)	
Δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
0	254	254	256	256	258	253	373.842	372.981	374.892	375.062	369.565	380.332
0.25	146	142	177	175	115	54	210.845	213.525	251.32	256.288	167.941	79.6229
0.5	83	84	118	119	31	15	118.577	120.723	166.455	166.304	42.4319	20.3075
0.75	49	48	80	78	9	5	68.6914	68.5462	114.619	116.531	12.3728	6.58483
1	29	29	54	53	3	2	41.0968	42	76.6202	78.8274	4.35318	2.45716
1.25	18	18	37	37	2	1	26.1511	25.6445	52.9877	54.6174	1.76417	1.07008
1.5	12	12	25	25	1	1	16.2487	16.2768	36.4437	36.4982	0.76672	0.4415
1.75	8	8	18	18	1	1	10.5237	10.1593	25.4012	25.8571	0.29544	0.14687
2	5	5	13	13	1	1	6.94577	7.09997	17.5475	17.6853	0.05821	0
2.25	4	4	9	9	1	1	4.72625	4.75848	12.5378	12.7902	0	0
2.5	3	3	7	7	1	1	3.38588	3.36977	8.88843	8.91622	0	0
2.75	2	2	5	5	1	1	2.39626	2.3811	6.36656	6.60131	0	0
3	2	2	4	4	1	1	1.73512	1.71333	4.65009	4.53701	0	0
3.25	1	1	3	3	1	1	1.28531	1.24803	3.31038	3.42979	0	0
3.5	1	1	2	2	1	1	0.95364	0.95271	2.49018	2.5042	0	0
3.75	1	1	2	2	1	1	0.7312	0.71925	1.80809	1.82018	0	0
4	1	1	1	1	1	1	0.53973	0.52382	1.34832	1.3229	0	0

Table III. (e) SDRL performance of Tukey and I/MR chart using exponential, Laplace, logistic, normal, and Rayleigh distributions

Distribution	Exponential (1)		Laplace (0, 1)		Logistic (6, 2)		Lognormal (0, .25)		Normal (0, 1)		Rayleigh (2)	
Δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
0	366.67137	373.84051	365.91871	372.99534	372.90428	373.56181	373.94871	369.83448	368.45334	376.6463	369.64225	370.7285
0.25	281.12617	287.04586	351.8927	355.90094	348.20191	336.15609	262.19438	245.3187	353.22403	292.24662	320.64472	209.77746
0.5	221.55912	219.17072	309.05086	299.43422	278.22801	263.68963	181.43663	166.66461	291.2011	156.59864	249.38795	127.96472
0.75	175.69005	177.19535	236.40019	227.292	205.44312	183.5606	123.00204	111.93732	208.26295	84.953201	174.56957	78.182852
1	133.44749	137.28579	169.61643	170.67978	138.87679	118.27841	81.947945	74.862247	129.38532	42.329727	112.2125	49.016082
1.25	103.73382	107.93847	125.73146	122.99933	92.147227	77.261292	54.497678	52.542712	74.708774	24.630575	68.92353	31.064691
1.5	80.991386	82.75496	92.700679	88.41594	59.841866	49.921862	37.287772	36.184461	42.164421	14.267357	43.44639	19.815218
1.75	65.125426	64.341382	61.561473	62.539682	38.72969	32.385672	26.240128	25.069008	24.640743	8.8737687	27.526528	13.297663
2	48.867616	49.894378	43.208769	43.136102	24.884869	20.358713	17.759417	16.792198	14.616663	5.9686279	18.225587	9.0537618
2.25	37.837996	38.387936	31.59711	29.708258	16.116075	13.290507	12.3904	11.75611	9.0433005	3.8984681	12.386583	6.462367
2.5	29.006398	30.675282	21.85795	21.170531	10.360408	8.3871977	8.7805156	8.3793082	5.6373778	2.730965	8.4533585	4.5572856
2.75	23.272519	23.699393	15.2094	14.924428	6.970618	5.5452165	6.2057444	5.9169636	3.9425625	1.9124972	5.9730834	3.2777116
3	17.423256	18.47031	10.655922	10.238997	4.4361347	3.73844	4.4487003	4.202754	2.7109397	1.4362271	4.2018976	2.4367471
3.25	13.915417	14.571573	7.1637216	7.0657193	2.9860285	2.5322277	3.3072041	2.986991	1.9508915	1.0902174	3.087893	1.7637795
3.5	10.727751	10.928996	4.8601013	4.7382602	2.0046812	1.6657402	2.2831315	2.1749464	1.4682517	0.8226267	2.2642738	1.3526147
3.75	8.0817796	8.1816062	3.2763714	3.1751111	1.4524504	1.2059107	1.6934875	1.5774708	1.0737118	0.6262284	1.7379679	0.9927994
4	6.2026946	6.5183114	2.10014	2.0314479	1.0718172	0.8975684	1.2217754	1.1459412	0.7920444	0.4746485	1.2556065	0.743128

Table III. (f) SDRL performance of Tukey and I/MR chart using binomial, geometric, Poisson, and Student's *t* distributions

Distribution	Binomial (50, .3)		Geometric (.3)		Poisson (10)		*t* (30)		*t* (10)		*t* (4)	
Δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
0	375.2462	374.9671	384.6194	451.8482	387.7642	383.4084	368.4838	378.4173	371.0544	366.37	379.458	368.7684
0.25	289.9097	241.849	297.4339	339.6826	239.9303	218.3973	336.4073	301.9237	342.9481	335.3299	360.2676	356.2523
0.5	193.8118	133.1271	217.7413	305.9369	129.5942	117.9442	276.6068	188.2094	284.8859	252.0384	298.6241	335.9418
0.75	116.8399	73.35631	181.2157	220.8213	69.54876	70.18644	187.9896	108.9705	222.78	171.8988	221.5824	300.8711
1	65.53731	39.41716	143.3483	188.2104	54.92091	42.69	117.5661	61.24316	158.7067	111.7643	170.9607	257.5804
1.25	37.90829	23.87062	103.5988	145.671	36.57478	25.95621	70.75625	34.86669	109.6369	68.736	126.1914	210.9178
1.5	22.51496	14.02946	81.08374	104.9288	20.4545	16.71987	41.47189	20.56721	68.84327	41.90901	88.4099	164.0208
1.75	13.47145	9.124814	67.63968	83.68937	11.49255	10.70732	24.37163	12.64954	43.09645	26.61243	62.02745	133.8156
2	8.579401	5.941207	49.22721	61.84107	7.002891	7.112901	14.97159	7.940952	27.02665	16.98235	43.89573	100.8533
2.25	5.702965	3.938829	38.52653	58.57827	6.172213	4.969179	9.413128	5.184349	17.02979	10.58546	30.3592	74.39231
2.5	3.7685	2.84368	32.28633	35.31778	4.143199	3.414852	6.037354	3.537302	10.98404	6.745632	21.39206	54.13142
2.75	2.673533	2.032092	23.7324	31.97855	2.737658	2.499691	4.067995	2.382903	7.056234	4.444892	14.62827	36.95699
3	1.953338	1.464703	17.64827	20.98737	1.842672	1.813159	2.746625	1.76648	4.748809	3.043396	10.23147	25.63547
3.25	1.445582	1.110188	14.60085	17.21436	1.274317	1.285875	1.984661	1.263124	3.107692	2.13478	7.232084	17.11522
3.5	1.087541	0.842452	11.36816	14.16955	1.158575	0.979822	1.414781	0.966774	2.147726	1.508201	4.765462	11.3827
3.75	0.815144	0.634882	7.978344	10.18663	0.840469	0.730615	1.069348	0.733448	1.543305	1.068065	3.196926	7.288847
4	0.620051	0.482941	6.163896	8.294542	0.589218	0.547288	0.776425	0.550198	1.104059	0.809155	2.046057	4.689889

Table III. (g) SDRL performance of Tukey and I/MR Chart using Gamma and Weibull distributions

Distribution	Gamma (4, 1)		Gamma (2, 1)		Gamma (1, 1)		Weibull (10, 1)		Weibull (3.5, 1)		Weibull (2, 1)	
Δ	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC	Xmr	TCC
0	369.29205	376.1841	369.29205	370.5177	365.61735	462.06872	369.16733	382.02696	376.0306	376.7669	376.1801	374.15254
0.25	248.83041	295.85471	248.83041	270.66984	282.94945	295.73148	594.4832	227.74397	203.874	198.0465	209.15485	211.92754
0.5	177.88234	205.83496	177.88234	199.75752	223.21844	225.09389	912.56661	86.127002	98.38997	90.9565	126.77073	128.54196
0.75	121.51532	135.93258	121.51532	145.17111	174.3795	176.97525	1208.4622	32.57664	47.23003	45.96202	77.390854	76.187826
1	82.541262	94.752469	82.541262	108.69461	137.9444	136.81842	1093.7699	14.438646	25.99329	23.94748	47.543989	48.686551
1.25	57.916958	66.530869	57.916958	78.587765	105.1709	105.91792	523.42905	7.67486	14.66845	13.95388	31.396384	31.649082
1.5	41.059568	45.792432	41.059568	57.912109	79.004446	82.036814	158.51976	4.5034985	9.003527	8.491934	20.028916	20.352081
1.75	28.536562	32.893018	28.536562	42.962212	63.105742	63.162741	54.70676	2.7954078	5.636357	5.566659	13.309996	13.471569
2	19.885597	22.870795	19.885597	32.141746	48.568519	49.709279	22.419527	1.8827221	3.764428	3.709633	9.1577653	9.0346899
2.25	14.55315	16.30594	14.55315	23.417675	37.784832	38.684221	10.956968	1.3981856	2.694394	2.59912	6.4309383	6.3952058
2.5	10.117057	11.481009	10.117057	17.376798	28.653873	30.331593	5.875805	1.0226967	1.903456	1.91107	4.624271	4.4767596
2.75	7.3892242	8.4513538	7.3892242	12.730013	23.067032	22.357399	3.6508283	0.7982356	1.43377	1.402663	3.3005388	3.3380066
3	5.5220346	6.1176403	5.5220346	9.4071531	17.498766	18.374644	2.3543706	0.631115	1.082536	1.088021	2.3725567	2.4615974
3.25	3.9637002	4.4427543	3.9637002	7.0359065	13.412431	14.229195	1.6127753	0.5073453	0.811738	0.79877	1.8048098	1.767372
3.5	2.8494135	3.2508027	2.8494135	5.1936105	10.705495	10.74455	1.1914204	0.3977711	0.61945	0.649737	1.3038485	1.3560729
3.75	2.0086203	2.3560794	2.0086203	3.9125689	7.9437763	8.1539614	0.9276678	0.3327951	0.493152	0.475948	1.0233003	1.0248548
4	1.4976803	1.7410475	1.4976803	2.8387628	6.0464673	6.261225	0.7188541	0.2680341	0.383027	0.371465	0.7404558	0.7406671

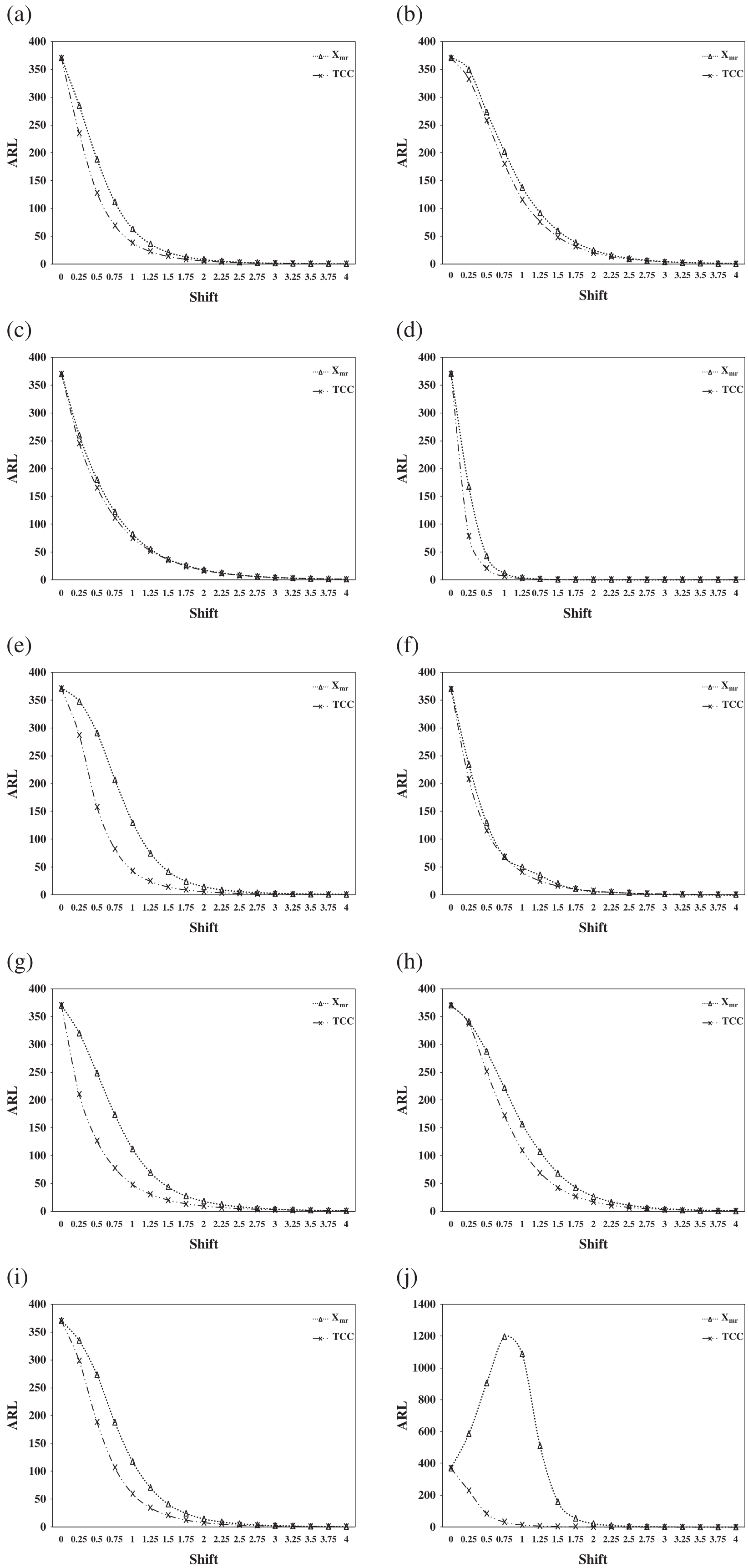

Figure 4. Tukey chart with higher efficiency than I/MR chart: (a) binomial (50, 0.3), (b) logistic (6, 2), (c) lognormal (0, 0.25), (d) Maxwell, (e) normal (0, 1), (f) Poisson (10), (g) Rayleigh; (h) *t* (10), (i) *t* (30), and (j) Weibull (10, 1) distribution

Table IV. (a) Tukey control chart multiplier using $ARL_0 \cong 168$ and $ARL_0 \cong 500$

Distribution	Exponential (1)	Laplace (0, 1)	Lognormal (0, .25)	Maxwell (2)	Normal (0, 1)	Rayleigh (2)
$ARL_0 \cong 168$	$K = 3.42345$ $K_U = 3.423$, $K_L = 2.4802$	$K = 3.21$	$K = 2.05$ $K_U = 2.05$, $K_L = 1.32122$	$K = 1.6324$	$K = 1.538783$	$K = 1.6895$ $K_U = 1.6895$, $K_L = 1.307$
$ARL_0 \cong 500$	$K = 4.4001$ $K_U = 4.4001$, $K_L = 3.28766$	$K = 3.99872$	$K = 2.564$ $K_U = 2.564$, $K_L = 1.8765$	$K = 1.9671$	$K = 1.79002$	$K = 2.0485$ $K_U = 2.04889$, $K_L = 1.3421$

Table IV. (b) I/MR control chart multiplier using $ARL_0 \cong 168$ and $ARL_0 \cong 500$

Distribution	Exponential (1)	Laplace (0, 1)	Lognormal (0, .25)	Maxwell (2)	Normal (0, 1)	Rayleigh (2)
$ARL_0 \cong 168$	$M_0 = 4.1452$ $R_0 = 5.678$	$M_0 = 3.63$ $R_0 = 6.031$	$M_0 = 3.2064$ $R_0 = 5.1272$	$M_0 = 3.1423$ $R_0 = 3.9852$	$M_0 = 3.41$ $R_0 = 3.87$	$M_0 = 3.184$ $R_0 = 4.023$
$ARL_0 \cong 500$	$M_0 = 5.232$ $R_0 = 7.8083$	$M_0 = 4.4025$ $R_0 = 7.387$	$M_0 = 3.8964$ $R_0 = 6.49959$	$M_0 = 3.82$ $R_0 = 4.4238$	$M_0 = 3.7801$ $R_0 = 4.383$	$M_0 = 3.784$ $R_0 = 4.503$

Table IV. (c) I/MR control chart multiplier using ($ARL_0 \cong 370$)

Distribution	Binomial (50, .3)	Chi-square (100)	Chi-square (10)	Exponential (1)	Gamma (4, 1)
$ARL_0 \cong 370$	$M_0 = 3.233$ $R_0 = 4.3801$	$M_0 = 3.1174$ $R_0 = 6.6131$	$M_0 = 3.789$ $R_0 = 6.8180$	$M_0 = 4.8969$ $R_0 = 7.4953$	$M_0 = 3.8943$ $R_0 = 6.431$
Distribution	Gamma (2, 1)	Geometric (0.3)	Laplace (0, 1)	Logistic (6, 2)	Lognormal (0, .25)
$ARL_0 \cong 370$	$M_0 = 4.3321$ $R_0 = 8.3081$	$M_0 = 4.911$ $R_0 = 6.32$	$M_0 = 4.1981$ $R_0 = 6.9301$	$M_0 = 3.76611$ $R_0 = 5.161$	$M_0 = 3.764$ $R_0 = 5.05882$
Distribution	Maxwell (2)	Normal (0, 1)	Poisson (10)	Rayleigh (2)	Student's *t* (30)
$ARL_0 \cong 370$	$M_0 = 4.1981$ $R_0 = 6.9301$	$M_0 = 3.5$ $R_0 = 4.2512$	$M_0 = 3.21$ $R_0 = 4.431$	$M_0 = 3.784$ $R_0 = 4.33$	$M_0 = 3.504$ $R_0 = 4.4211$
Distribution	Student's *t* (10)	Student's *t* (4)	Weibull (10, 1)	Weibull (3.5, 1)	Weibull (2, 1)
$ARL_0 \cong 370$	$M_0 = 3.803$ $R_0 = 4.8101$	$M_0 = 4.5783$ $R_0 = 6.91$	$M_0 = 3.482$ $R_0 = 5.187$	$M_0 = 2.7562$ $R_0 = 4.732$	$M_0 = 3.3362$ $R_0 = 5.991$

Table IV. (d) Tukey control chart multiplier using ($ARL_0 \cong 370$)

Distribution	Binomial (50, 0.3)	Chi-square (100)	Chi-square (10)	Exponential (1)	Gamma (4, 1)
$ARL_0 \cong 370$	$K = 1.9342$	$K = 1.834$ $K_u = 1.834$, $K_L = .9888$	$K = 2.4804$ $K_u = 2.4804$, $K_L = 1.2518$	$K = 4.121$ $K_u = 4.121$, $K_L = .262$	$K = 2.594$ $K_u = 2.667$, $K_L = .859$
Distribution	Gamma (2, 1)	Geometric (0.3)	Laplace (0, 1)	Logistic (6, 2)	Lognormal (0, .25)
$ARL_0 \cong 370$	$K = 3.138$ $K_u = 3.138$, $K_L = .555$	$K = 4.323$	$K = 3.76681$	$K = 2.507045$	$K = 2.4251$ $K_u = 2.4565$, $K_L = 1.30001$
Distribution	Maxwell (2)	Normal (0, 1)	Poisson (10)	Rayleigh (2)	Student's *t*(30)
$ARL_0 \cong 370$	$K = 1.8741$	$K = 1.7238$	$K = 1.965321$	$K = 1.955$ $K_u = 1.95472$, $K_L = .999$	$K = 1.895$
Distribution	Student's *t* (10)	Student's *t* (4)	Weibull (10, 1)	Weibull (3.5, 1)	Weibull (2, 1)
$ARL_0 \cong 370$	$K = 2.327$	$K = 3.969$	$K = 2.190$ $K_u = 1.239$, $K_L = 2.298$	$K = 1.466$ $K_u = 1.468$, $K_L = 1.462$	$K = 1.957$ $K_u = 2.121$, $K_L = .785$

Other performance measures, EQL, RARL, and PCI, are reported in Tables II(a)–II(c). It is further observed that both control charts have similar performance. XmR was more efficient in the case of Student's *t* (4 degrees of freedom) and gamma (4, 1) distributions (Figure 3).

Similar conclusions are reached based on other performance measures (Tables I(a)–III(g)). TCC had a more efficient ARL, when the data were distributed from binomial, logistic, lognormal, Maxwell, normal, Poisson, Rayleigh, Student's *t* (with 30 and 10 degrees of freedom), Weibull (10, 1), and Weibull (3.5, 1). ARL curves are presented in Figure 4.

Other performance measures, EQL, RARL and PCI, are reported in Tables II(a)–II(c). Similar conclusions are reached whether we rely on ARL or other measures of performance. TCC had better EQL, RARL, PCI, SDRL, and MDRL measures than the XmR chart.

It is to be mentioned that the control limit multipliers for both the charts under discussion are evaluated for different choices of ARL_0. We have provided these multipliers in Tables IV(a)–IV(d) at ARL_0 of 168, 370, and 500 and different probability distributions.

5. Conclusions and recommendations

Both TCC and XmR chart were designed to be simple to use and require no extensive access to computers or calculations. Neither one requires the calculation of standard deviation: a procedure that is difficult if one does not have access to computers. Both methods make limited assumptions about the nature of the data and can be widely used. Both TCC and XmR chart performed well in the analysis of different simulated data. Improvement teams have a choice to make about which control chart best suits their needs. In health care, early detection of adverse events (e.g., suicide, onset of cancer, medication errors, etc.) is important. TCC and XmR differ in their speed of detecting a shift in a process.

There are some situations in which both control charts have similar performance. There are also many instances where TCC was preferred to XmR and a few where the reverse occurred. TCC was more efficient, when the data came from binomial, Rayleigh, Maxwell, normal, logistic, lognormal, Weibull, and Student's *t* (with 30 and 10 degrees of freedom) distributions. One would expect many processes to follow these distributions. Normal distribution is quite common. Because these distributions are common, one would infer that in most common situations, TCC is more or equally accurate than XmR.

Most improvement teams are not sure about the distribution of their data. TCC is the better choice as it outperforms XmR in normal and many nonnormal environments. Future research should examine the performance of TCC and XmR charts in situations where cumulative sum is calculated and events in recent time periods are weighted more heavily than events in older time periods. Another option may be a memory structure in the form of exponentially weighted moving average charts for an efficient detection of smaller shifts.

Acknowledgements

The authors are thankful to the reviewer(s) for the constructive comments that helped in improving the last version of the paper. The authors are also grateful to Dr. Shabbir Ahmad (Assistant Professor at Commission on Science and Technology for Sustainable Development in the South Wah campus) for his intellectual support in improving the initial version. The author Muhammad Riaz is also indebted to King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia, for providing excellent research facilities.

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