**Chapter 9**

# **Analysis of One Observation per Time Period: Tukey Chart****[H1] Learning Objectives**

**[INSERT NL]**

1. Define distribution of medians
2. Calculate quartiles
3. Create Tukey control limits

**[END NL]**

# **[H1] Key Concepts**

**[INSERT BL]**

* Tukey control limits for medians
* Interquartile difference

**[END BL]**

# **[H1] Chapter at a Glance**

Up until chapter 9, this book has examined the analysis of means and rates, which assumes that each time period has several observations. In this chapter, we will assume that we have only one observation per period and no good knowledge of the distribution of these data. Analysis of one observation per period seems impossible—after all, what is there to analyze? We have only a single observation. The solution to this problem arises when we relax our concept of time periods, thereby gaining more observations. For example, we can calculate the moving average over four periods, or we can calculate the probability of an event over a longer time. In this chapter, we use the median of values over time to test whether observations are changing over time. By dividing the sample into medians and fourths (the median of the median), we have a method for overcoming our insufficient observations per period.

# **[H1] Tukey’s Chart**

We will use Tukey’s suggested confidence interval limits for the calculation of control limits (Alemi 2004; Borkardt et al. 2005). The procedure for calculating control limits is to calculate the difference between the upper fourth and the lower fourth of the data, a concept that mathematician John Tukey named the *fourth spread.* Most readers are familiar with the idea of the median—a value of which half the data are below and half the data are above. A lower fourth is similar to the 25 percent quartile, and the specific number is the median of the first half of the data. At this point, 25 percent of the data are below this value. An upper fourth is similar to the 75 percent quartile and is the median of the upper half of the data; at this point, 75 percent of the data are below this value. The difference between the two fourths is referred to as the *fourth spread*. The upper control limit (UCL) is calculated as the sum of the upper fourth and 1.5 times the fourth spread. The lower control limit (LCL) is calculated as the difference between the lower fourth and 1.5 times the fourth spread.

The procedure for calculating Tukey’s control limits has five steps:

**[INSERT NL]**

1. List the values of the data from smallest to largest.
2. Calculate the median. If the number of observations is odd, use the value in the middle. If there is an even number of observations, take the average of the two middle-ranked numbers.
3. Divide the data set into two halves using the median. Include the median in both halves if the median is one of the observed data points.
4. The lower fourth data point is the median of the lowest 50 percent of the data—data from the smallest number up to (or including) the median.
5. The upper fourth is the median of the top 50 percent of the data—numbers ranging from (or including) the median of the full data set to the highest value.
6. Calculate the fourth spread as the difference between the two fourths.
7. Calculate the UCL and LCL using the following two formulas:

**[INSERT EQUATION]**  
LCL = Lower fourth − 1.5 × Fourth spread, and

UCL = Upper fourth + 1.5 × Fourth spread.

**[END EQUATION]**

**[END NL]**

## **[H1] Example 1: Time to Pain Medication**

The Hospital Compare website of Medicare.gov (CMS) (Centers for Medicare & Medicaid Services 2017) provides information on the performance of individual hospitals. Numerous measures are provided. This example uses data provided on the OP\_21 measure, often referred to as time to pain medication for patients with a diagnosis of a long bone fracture. The Agency for Healthcare Research and Quality (AHQR) provides the following [rationale](https://www.qualitymeasures.ahrq.gov/summaries/summary/49600) for why this measure is important:

**[INSERT BLOCK QUOTE]**

This measure is used to assess the time (in minutes) from emergency department arrival to time of initial oral, intranasal or parenteral pain medication administration for ED patients with a principal diagnosis of long bone fracture. Pain management in patients with long bone fractures is undertreated in emergency departments. Emergency department pain management has room for improvement. Patients with bone fractures continue to lack administration of pain medication as part of treatment regimens. When performance measures are implemented for pain management of these patients, administration and treatment rates for pain improve. Disparities continue to exist in the administration of pain medication for minorities and children as well. (Agency for Healthcare Research and Quality 2018)

**[END BLOCK QUOTE]**

Poor performance on this measure may result from many causes. One way to visualize this measure is to consider the experience of a mom frantically driving her child to the emergency department (ED). The child has a painful broken leg, and the mother and the child are waiting in the ED for treatment. In 2016, the median time to pain medication on this measure was 52 minutes. In other words, in 50 percent of EDs, patients waited more than 52 minutes before they received pain medication for their broken bones.

Of course, there are multiple explanations for long waits that are unrelated to ED efficiency. For example, only patients cared for in EDs and discharged to home are included, excluding the most serious injuries (e.g., multiple-trauma car accidents) that require subsequent hospitalization. Data do not include patients who received pain medication en route to the ED. It is also possible that patients may have received undocumented pain medication during ED visits. Patients may also have been offered pain medication but refused, with their refusal unrecorded. Despite the flaws in the OP\_21 ED performance measure, it is still a reasonable measure for a subset of patients. Hospitals that do poorly on this measure should either improve their record keeping or improve their response time to ED patients, both of which are important. Many healthcare managers are sensitive to improving these measures.

This example features the performance of **Southeast Alabama Medical Center** over two years. CMS uses code 010001 for this provider. The Hospital Compare website reports the measure OP\_21 for all providers using two fields: score and sample. The score is minutes from ED arrival to time of initial oral, intranasal, or parenteral pain medication administration. The sample is the number of patients used to calculate the minutes to pain medication. In our analysis, we focus on the score variable.

The first step in the analysis is to download the Hospital Compare files for the years 2015 and 2016. Because of a lag in reporting, these years have data from 2013 to 2015. Hospital Compare data files contain many tables. In these files, we select table HQI\_HOSP\_TimelyEffectiveCare. Once the table for each period has been downloaded, we repeatedly run the following code to extract the fields of interest from the tables:

**[LIST FORMAT]**

SELECT

HQI\_HOSP\_TimelyEffectiveCare\_2015\_01.[Measure ID], HQI\_HOSP\_TimelyEffectiveCare\_2015\_01.[Provider ID], HQI\_HOSP\_TimelyEffectiveCare\_2015\_01.[Measure Start Date], HQI\_HOSP\_TimelyEffectiveCare\_2015\_01.[Measure End Date], HQI\_HOSP\_TimelyEffectiveCare\_2015\_01.Score, HQI\_HOSP\_TimelyEffectiveCare\_2015\_01.Sample

FROM HQI\_HOSP\_TimelyEffectiveCare\_2015\_01

WHERE (HQI\_HOSP\_TimelyEffectiveCare\_2015\_01.[Measure ID]="OP\_21") AND (HQI\_HOSP\_TimelyEffectiveCare\_2015\_01.[Provider ID]="010001");

**[END LIST]**

This structured query language (SQL) code selects measure ID OP\_21 and provider ID 010001. Making these selections across the various data files allows us to examine the data reported in exhibit 9.1. Note that the data files released in the fourth and fifth months of 2015 refer to the same start and end dates, so one of these reports is duplicative and can be ignored.

**[INSERT EXHIBIT]**

**Exhibit 9.1** Time to Pain Medication for Long-Bone Fracture in Emergency Department

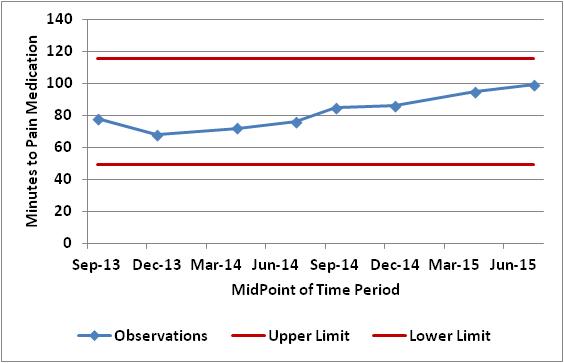
|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *Measure ID* | *Provider ID* | *Measure Start Date* | *Measure End Date* | *Score* | *Sample* | *From Database* |
| OP\_21 | 010001 | 04/01/2013 | 03/31/2014 | 78 | 132 | [2015-01-22.zip►](http://medicare.gov/download/HospitalCompare/2015/January/HOSArchive_20150122.zip) |
| OP\_21 | 010001 | 07/01/2013 | 06/30/2014 | 68 | 124 | [2015-04-16.zip►](http://medicare.gov/download/HospitalCompare/2015/April/HOSArchive_20150416.zip) |
| OP\_21 | 010001 | 07/01/2013 | 06/30/2014 | 68 | 124 | [2015-05-06.zip►](http://medicare.gov/download/HospitalCompare/2015/May/HOSArchive_20150506.zip) |
| OP\_21 | 010001 | 10/01/2013 | 09/30/2014 | 72 | 122 | 2015-07-16.zip[►](http://medicare.gov/download/HospitalCompare/2015/July/HOSArchive_20150716.zip) |
| OP\_21 | 010001 | 01/01/2014 | 12/31/2014 | 76 | 138 | [2015-10-08.zip►](http://medicare.gov/download/HospitalCompare/2015/October/HOSArchive_20151008.zip) |
| OP\_21 | 010001 | 04/01/2014 | 03/31/2015 | 85 | 144 | 2015-12-10.zip► |
| OP\_21 | 010001 | 07/01/2014 | 06/30/2015 | 86 | 137 | [2016-05-04.zip►](https://medicare.gov/download/HospitalCompare/2016/May/HOSArchive_20160504.zip) |
| OP\_21 | 010001 | 10/01/2014 | 09/30/2015 | 95 | 121 | [2016-08-10.zip►](https://www.medicare.gov/download/HospitalCompare/2016/August/HOSArchive_20160810.zip) |
| OP\_21 | 010001 | 01/01/2015 | 12/31/2015 | 99 | 111 | [2016-11-10.zip►](https://medicare.gov/download/HospitalCompare/2016/October/Hospital_20161110.zip) |

**[END EXHIBIT]**

To construct a Tukey control chart, we follow the steps outlined for calculating control limits earlier in this chapter. In the first step, we list scores from small to large: 68, 72, 76, 78, 85, 86, 95, and 99. The median of these values can be calculated using the median function in Excel; using that, we arrive at 81.5. There is an even number of observations, so the average between the two middle-ranked numbers constitutes the median. This median divides the data into two halves. The first half comprises four numbers: 68, 72, 76, and 78. The second half is the rest of the observations. The median of the values less than 81.5 is 74; it is called the *lower fourth*. The median of the values above 81.5 is 90.5; it is called the *upper fourth*. The *fourth spread* is 90.5 − 74 = 16.5. The UCL is the upper fourth plus 1.5 times the fourth spread, which is 81.5 + 1.5 × 16.5 = 115.25. The LCL is the lower fourth minus 1.5 times the fourth spread, which is 74 − 1.5 × 16.5 = 49.25. Exhibit 9.2 shows the Tukey control chart for time to pain medication for this hospital. The control chart in exhibit 9.2 shows us that the values do not exceed the control limits. This tells us that these values could result from random chance, though there has been a steady increase in time to pain medication starting at the second point. The ED time to pain medication has not changed over this period.

**[INSERT EXHIBIT; labels on axes should be rom, not bold. Render in gray scale. Make upper red line dashes and label it UCL. Make lower red line dots and label is LCL. Make center line solid black.]**

**Exhibit 9.2 Time to Pain Medications for Southwest Alabama Medical Center’s Emergency Department**



## **[END EXHIBIT]**

## **[H1] Example 2: Exercise Time and Weight Loss**

Jane collected data as described in exhibit 9.3 regarding her exercise times. She planned to exercise three times a week, and each time she exercised, she recorded the time in minutes. When she did not exercise, she recorded a 0 for the length of exercise. The first seven days recorded were pre-intervention. After this period, she and her spouse joined a volleyball team, and she wanted to know whether joining the team had made a difference in her exercise time.

**[INSERT EXHIBIT]**

**Exhibit 9.3** Length of Exercise

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | *Initial Unsorted Data* | |  | *Sorted in Order of Length of Exercise* | | |
| *Day* | *Minutes of Exercise* |  | *Rank* | *Day* | *Minutes of Exercise* |
| Pre-intervention | 1 | 30 |  | 1 | 2 | 0 |
| 2 | 0 |  | 2 | 3 | 25 |
| 3 | 25 |  | 3 | 1 | 30 |
| 4 | 30 |  | 4 | 4 | 30 |
| 5 | 35 |  | 5 | 5 | 35 |
| 6 | 40 |  | 6 | 6 | 40 |
| 7 | 50 |  | 7 | 7 | 50 |
| Post-intervention | 8 | 45 |  |  |  |  |
| 9 | 31 |  |  |  |  |
| 10 | 20 |  |  |  |  |
| 11 | 40 |  |  |  |  |
| 12 | 60 |  |  |  |  |
| 13 | 45 |  |  |  |  |
| 14 | 60 |  |  |  |  |
| 15 | 45 |  |  |  |  |
| 16 | 32 |  |  |  |  |
| 17 | 50 |  |  |  |  |
| 18 | 60 |  |  |  |  |

**[END EXHIBIT]**

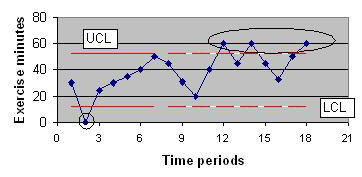
We can calculate the control limits from the pre-intervention or post-intervention periods and select the limits that are tighter (meaning the difference between the UCL and the LCL is smaller). It so happens that the limits calculated from the pre-intervention period are tighter, making it the better basis for calculation.

The first step is to sort pre-intervention data in order of length of exercise. This is shown in exhibit 9.3, in the last column of the table. Next, we calculate the median—the point at which one-half of the data (7 × .5 = 3.5, 3 points) is lower and one-half of the data (3 points) is above it. The fourth data point with a value of 30 is the median; three data points are below it and three above it. Because the median is an actual data point, we include this point in the lower data set. To calculate the lower fourth, we calculate the halfway point for the first half of the data. When we include the median, we have four points in the lower data set. The 25 percent quartile is halfway between the second and third points—in other words, between 25 and 30, which is 27.5.

To calculate the upper fourth, we calculate the halfway point for the upper half of the data. Again, because the median is an actual data point, we include this point in the upper data set. With the median, we have four data points from the median to the highest values. The upper fourth is between the fifth and sixth data points (between 35 and 40), and therefore its value is 37.5. The fourth spread is the difference between the upper and lower fourth, which is  
37.5 − 27.5 = 10. The fourth spread for the control limits calculated from post-intervention data is 18 points, which is why we will calculate control limits from pre-intervention data. The UCL is calculated as 37.5 + 1.5 × 10 = 52.5. The LCL is calculated as 27.5 − 1.5 × 10 = 12.5. A chart of the data is provided in exhibit 9.4.

**[INSERT EXHIBIT; make image larger, please. Render in gray scale. Eliminate gray background (make transparent). Make upper red line dashes (then, to the right, smaller dashes), and label it UCL (but without that white box around it). Make lower red line dots (then, on the right, smaller dots), and label it LCL (but without the white box around it). Make blue line solid black. Make axis labels rom, not bold.]**

**Exhibit 9.4** Tukey's Control Chart for Data in Exhibit 9.3



**[END EXHIBIT]**

Examination of the chart shows that in the first seven days, there was one very low point of no exercise. After the first seven days (used for setting the limits), on three occasions the total exercise time exceeded the UCL. In these three days, there was a real increase in exercise time compared to the first seven days. If these days correspond to joining the volleyball team, then the intervention seems to have worked.

In interpreting control charts, it is important not to overstate the case. The fact that some points fall outside the control limits does not mean that a lasting change has occurred. Only the points outside the control limits are statistically significant relative to pre-intervention periods. In exhibit 9.4, three points are outside the control limit, but many other points are within. A statement that Jane is exercising more may be overstating the case; Jane has exercised more in 3 periods out of 11 post-intervention periods.

## **[H1] Example 3: Keeping Exercise Patterns**

Let us look at another example, this time related to weight loss. A 48-year-old man measured his weight for eight weeks. Then he and his spouse changed food-shopping habits. They excluded all sweets from their shopping (they stopped buying sodas, sweetened cereals, and chocolates). The data for this person is provided in exhibit 9.5. Weight was recorded once a week.

**[INSERT EXHIBIT]**

**Exhibit 9.5** Recorded Weight Values

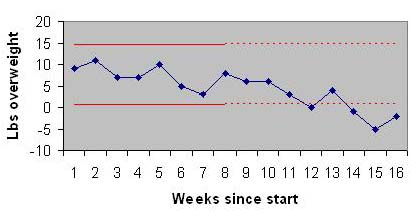
|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *Unsorted Observations* | |  | *Sorted Values* | |
| *Week* | *Pounds More Than Ideal Weight* |  | *Rank* | *Pounds More Than Ideal Weight* |
| Pre-intervention | 1 | 9 |  | 1 | 3 |
| 2 | 11 |  | 2 | 5 |
| 3 | 7 |  | 3 | 7 |
| 4 | 7 |  | 4 | 7 |
| 5 | 10 |  | 5 | 8 |
| 6 | 5 |  | 6 | 9 |
| 7 | 3 |  | 7 | 10 |
| 8 | 8 |  | 8 | 11 |
| Post-intervention | 9 | 6 |  |  |  |
| 10 | 6 |  |  |  |
| 11 | 3 |  |  |  |
| 12 | 0 |  |  |  |
| 13 | 4 |  |  |  |
| 14 | −1 |  |  |  |
| 15 | −5 |  |  |  |
| 16 | −2 |  |  |  |

**[END EQUATION]**

As before, we need to calculate the control limits for the pre- and post-intervention periods and select the limits with the smallest difference. It so happens that that the pre‑intervention period has the smallest fourth spread. The first step is to sort pre-intervention data from the lowest to the highest number of pounds. This is shown in exhibit 9.5, in the last column of the table. Next, we calculate the median—the value at which one-half of the data  
(8 × .5 = 4 points) is below it and one-half of the data (4 points) is above it. The value should be between the fourth and fifth data points, or between 7 and 8, so the median is 7.5.

Because a median is not an actual data point, we do not include this point in the calculations of fourths. To calculate the lower fourth, we pick the halfway point for the first half. We have four points in the lower data set. The lower fourth is halfway between the second and third points—in other words, between 5 and 7, it is 6. To calculate the upper fourth, we find the halfway point for the upper half of the data. Again, because the median was not an actual data point, we do not include this point in the upper data set. We have four data points for the highest values. The upper fourth is between the sixth and seventh data points (between 9 and 10), and therefore it is 9.5. The fourth spread is the difference between the upper and lower fourth, which is 9.5 − 6 = 3.5. The UCL is calculated as 9.5 + 1.5 × 3.5 = 14.75. The LCL is calculated as 6 − 1.5 × 3.5 = 0.75. A chart of the data is provided in exhibit 9.6.

**[INSERT EXHIBIT; make image larger, please. Render in gray scale. Eliminate gray background (make transparent). Make upper red line dashes (then, to the right, smaller dashes), and label it UCL. Make lower red line dots (then, on the right, smaller dots), and label it LCL. Make blue line solid black. Make axis labels rom, not bold.]**  
**Exhibit 9.6** Control Chart for the Weight Data



**[END EXHIBIT]**

Examination of the chart shows that in the first eight weeks, all data points were within the limit. No weight was lost in the pre-intervention period, even though the man experienced considerable fluctuation. Compared to the first eight weeks, his weight was lower than the LCL on four occasions during the last eight. Therefore, he may have seen a real decrease in weight in the post-intervention period. Keep in mind that in all control charts, we compare the observed values to some reference data; in the case of exhibit 9.6, the reference is the pre-intervention data.

## **[H1] Example 4: Medication Errors**

The following data show an error in a hospital’s medication dispensation system. To improve access to the medication, a machine is located at the nurses’ station so that the nurses can have access to the commonly used medication without requesting it from a pharmacy, which is located several floors or buildings away. Nurses enter information about the needed medication into the machine and the machine releases the medication on demand. Unfortunately, some errors may occur as the machine is refilled. The data in exhibit 9.7 show the date of these errors for one hospital in a two-month period.

**[INSERT EXHIBIT]**

**Exhibit 9.7** Recent Refill Errors

|  |  |  |
| --- | --- | --- |
| *Name of Drug Refilled* | *Date of Error* | *Type of Error* |
| Promethazine inj 25 mg | 05/13/07 | Wrong strength |
| Lidocaine 1% with Epi | 06/13/07 | Wrong strength |
| Ofloxocin Opth | 05/14/07 | Wrong dose form |
| Sensorcaine 0.25% | 05/07/07 | Wrong strength |
| Lidocaine 1% with Epi | 06/20/07 | Wrong strength |
| Tramadol 50 mg tab | 05/21/07 | Wrong drug |
| Morphine sulfate 30 mg | 07/01/07 | Wrong dosage |

**[END EXHIBIT]**

Note that there are no pre- or post-intervention periods; therefore, the entire data set can be used to calculate control limits. To analyze these data, we first calculate the time between errors (exhibit 9.8). The median for the data is 7. The upper fourth is the median of 7, 11, and 23, which is 11. The lower fourth is the median of 1, 6, and 7, which is 6. The fourth spread is five days. The UCL is the upper fourth plus 1.5 times the fourth spread—18.5. The LCL is a negative number, so it is reset to zero. Exhibit 9.9 shows the control chart. These data suggest that June 13 was different from the other five periods, perhaps suggesting a different pharmacist was on-site.

**[INSERT EXHIBIT]**

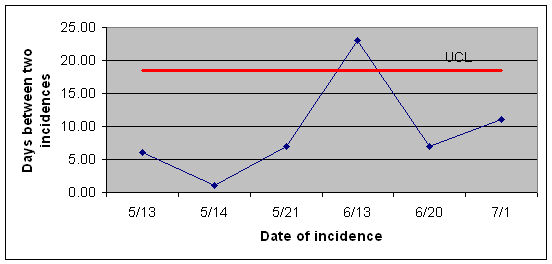
**Exhibit 9.8** Days to Errors

|  |  |
| --- | --- |
| *Date of Ending Incident* | *Days Between* |
| 5/13 | 6 |
| 5/14 | 1 |
| 5/21 | 7 |
| 6/13 | 23 |
| 6/20 | 7 |
| 7/1 | 11 |

**[END EXHIBIT]**

**[INSERT EXHIBIT; make image larger, please. Render in gray scale. Eliminate gray background (make transparent). Make upper red line dashes Make blue line solid black. Make axis labels rom, not bold.]**

**Exhibit 9.9** Medication-Dispensing Machine Refill Errors



## **[END EXHIBIT]**

## **[H1] Example 5: Budget Variation**

Tukey charts can also be used to compare *observed* to *expected* values. Suppose that we are looking at 12 months of data regarding our clinic’s budget. The question is whether the expenditures during any particular month are higher than the general pattern across the 12 months. Exhibit 9.10 shows the budget deviation (expenditure minus budget amount) for each of the months in thousands of dollars.

**[INSERT EXHIBIT]**

**Exhibit 9.10** Budget Deviations

|  |  |
| --- | --- |
| *Month* | *Budget Deviation (in $1,000s)* |
| 1 | 23 |
| 2 | −5 |
| 3 | −70 |
| 4 | −7 |
| 5 | −8 |
| 6 | 9 |
| 7 | 12 |
| 8 | 30 |
| 9 | 24 |
| 10 | 25 |
| 11 | −4 |
| 12 | −2 |

**[END EXHIBIT]**

Note that there are no pre- and post-intervention periods and therefore the entire data set is used for the estimation of control limits. The first step is to sort the data, as we see in exhibit 9.11.

**[INSERT EXHIBIT]**

**Exhibit 9.11** Sorted Budget Deviations

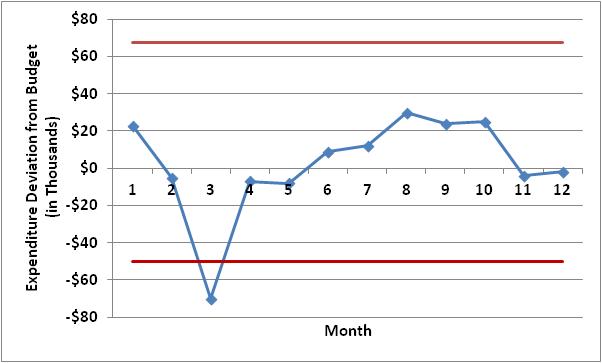
|  |  |
| --- | --- |
| *Rank* | *Budget Deviation (in $1,000s)* |
| 1 | −70 |
| 2 | −8 |
| 3 | −7 |
| 4 | −5 |
| 5 | −4 |
| 6 | −2 |
| 7 | 9 |
| 8 | 12 |
| 9 | 23 |
| 10 | 24 |
| 11 | 25 |
| 12 | 30 |

**[END EXHIBIT]**

There are 12 data points, so the median is halfway between the sixth and seventh ranked data points. Therefore, the median is not included in the lower and upper data sets because it is not an actual value in the data. The lower fourth is halfway between the 6 data points with lowest ranks; it is between the third- and fourth-ranked data points and has a value of −6. The upper data set is the points ranked 7 through 12. The median of this data set is halfway in between the ninth- and tenth-ranked data items. It is 23.5. The fourth spread is 29.5. The UCL is 23.5 + 1.5 × 29.5 = 67.75, and the LCL is −6 − 1.5 × 29.5 = −50.25. Exhibit 9.12 shows the associated control chart. The chart shows that all months are within the control limits, except for the month of March. In March, expenditures exceeded the budgeted amount, and the deviation was not random but marked a pattern distinct from previous periods.

**[INSERT EXHIBIT; render in gray scale. Eliminate gray background (make transparent). Make upper red line dashes, and label it UCL. Make lower red line dots and label it LCL. Make blue line solid black. Replace hyphens before negative numbers with actual minus signs. Make axis labels and numbers rom, not bold.]**

**Exhibit 9.12** Tukey Chart for Budget Information (in $1,000)



**[END EXHIBIT]**

In interpreting a control chart, we rely on the source of the control limits to clarify what is the reference point. If control limits are derived from historical patterns, we compare the date to historical patterns. If control limits were calculated from expected patterns (e.g., risk-adjusted patterns), the comparison group is the pattern expected from patients’ risks. We always refer to a specific comparator. Exhibit 9.12 compares observed expenditures to budgeted amounts. The reference is not historical patterns but expected observations, as set by the budget.

# **[H1] Comparison of Tukey and Other Charts**

Many different types of charts can be used to track healthcare data. P-charts are useful for analysis of mortality data but assume large number of observations per period. X-bar charts are useful for analysis of satisfaction ratings but assume normal data and multiple observations per period. When there is only one observation per period, three charts can be used to present data: XmR, Tukey, and time-between charts. Time-between control charts are for dichotomous events that occur once per period. Both XmR and Tukey charts assume a continuous measure. XmR charts were previously explained in chapter 6. The following section explains why displaying data with Tukey charts can be preferable to commonly used XmR charts.

Tukey’s charts are robust, in the sense that outliers do not affect the control limits. In XmR control charts, outliers can make control limits become wider. In the Tukey chart, this does not occur—its calculation of medians and medians is not affected by outliers. Studies that have compared Tukey and XmR have generally concluded that Tukey charts are more efficient than XmR charts in many circumstances, including when the observed data has “binomial, Rayleigh, logistic, lognormal, Maxwell, normal, Poisson, Weibull (with α = 10, β = 1), and Student’s *t* (30 and 10 degrees of freedom) distributions” (Khaliq, Riaz, and Alemi 2015, 1063). For a wide set of distributions, Tukey charts are more accurate than XmR charts. The only two distributions for which XmR is more efficient are in Student’s *t*-distribution with four degrees of freedom and gamma distribution with α = 4, β = 1. This book has not covered the gamma distribution, but it will suffice to know that it is a two-parameter family of continuous probability distributions; a special case of this distribution includes the more familiar exponential distribution. Others have also reported that the Tukey chart does not do well in populations with gamma distributions (Torng et al. 2009). Gamma distributions are not rare and do occur in many healthcare situations, so it is important to check the distribution of the data before choosing between XmR and Tukey charts.

In 2012, scholar Victor Tercero-Gomez and colleagues modified the Tukey chart to improve the insensitivity of this approach to gamma distributions. These scholars changed the 1.5 parameter used in calculating control limits in the Tukey chart. The modified Tukey was sensitive to shifts of the means in skewed distributions such as gamma distributions (Lee and Torng 2016). Others have also experimented with assigning different parameters to the UCL and UCL of the Tukey chart, resulting in further improvement in accuracy (Lee 2011; Sukparungsee 2013). From these investigations, we can conclude that for relatively symmetric data, the constant 1.5 remains appropriate; in many circumstances, this simple charting method is superior to XmR charts. For skewed distributions, such as exponential or gamma distribution, other constants should be used.

# **[H1] Summary**

This chapter introduced Tukey charts, which are designed for the analysis of a repeated single measure over time.

# **[H1] Supplemental Resources**

Problem set, solutions to problems, multimedia presentations, SQL code, and other related materials are in the course website.**[H1]** **References**

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