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Tutorial on Discrete Hazard Functions

Farrokh Alemi, PhD

Risk analysis requires estimation of hazard functions. A hazard rate is the conditional probability of adverse sentinel event occurring in the next time period, given that it has not yet occurred. This tutorial shows how hazard functions are estimated from survival functions, the probability of going through a time period without the sentinel event. Survival functions are built on cumulative distribution functions, which measure the probability of occurrence of sentinel event in current and prior time periods. Cumulative distribution functions are calculated from probability density functions, which give the probability of an event occurring at a particular time period. Probability density functions are typically estimated from incidence reports, which are readily available to safety officers. Sometimes, these functions are estimated by making assumptions about the shape of the distribution function. For discrete data, the typical probability density functions are Bernoulli, Binominal, Geometric, and Poisson distributions. This tutorial starts with estimating a probability distribution and then proceeds to calculation of hazard and relative risk rates.

Key words: *attributable risk, Bernoulli, binomial, discrete probability, geometric, hazard function, Poisson*

This tutorial is part of a series of articles intended to prepare the reader to apply probability concepts to analysis of adverse sentinel events.[1] The analysis of sentinel events is marked with paucity of data. Adverse events are rare. So, the method of analyzing these data must assume few observations. Since most adverse events are discrete events, they either happen or do not happen; the analysis is further restricted to discrete data. These limitations make analysis of incidence of adverse events difficult. Biostatisticians have, for sometime, developed methods of analyzing binary data. These methods are typically known as actuarial analysis and are in common use by engineers, actuaries, and a number of other professions. In 2002, Cox provided a very thorough explanation of risk analysis using discrete data.[2] This tutorial takes the methods discussed by Cox and shows how they apply to analysis of risks of sentinel events. Readers are encouraged to consult the 2002 book by Cox as well as several other books on risk analysis and analysis of binary data.[3,4]

This tutorial starts with estimating a probability density function for sentinel events from incidence reports and then proceeds to calculation of hazard and relative risk rates associated with different causes of sentinel events. While many safety officers will use computer software for calculation of hazard rates, this tutorial is intended to give them a sense of the logic behind the computer operations.

PROBABILITY DENSITY FUNCTIONS

A function assigns numbers to events. A probability density function gives the frequency of occurrence

Author Affiliation: *Health Administration and Policy, College of Health and Human Services, George Mason University, Fairfax, Virginia.*

Corresponding Author: *Farrokh Alemi, PhD, Health Administration and Policy, College of Health and Human Services, George Mason University, 4400 University Dr, Fairfax, VA 22030 (falemi@gmu.edu).*

Table 1

EXAMPLE OF PROBABILITY DENSITY AND CUMULATIVE DISTRIBUTION FUNCTIONS

Day	Probability density function	Cumulative distribution function
1	0.01	0.01
2	0.06	0.07
3	0.04	0.11
4–365	0.89	1

of events at a particular time. In contrast, a cumulative probability distribution function gives the probability of events occurring prior to a particular time.

Take a look at Table 1. For each day, the number of medication errors is divided by the number of patients to see what the daily probability of medication errors is. The data were collected over a year but for simplicity, we have collapsed days 4 through 365 into 1 category. The first column shows the days, and the second column shows the probability density function. At each value, it provides the probability of the event listed. For example, 1% of medication errors occur in day 1 (or the probability of medication error in day 1 is 0.001), 6% occur in day 2, 4% occur in day 3, and the remainder of the errors occurs in days 4 through 365. The cumulative distribution function is also given in the third column. It starts at 1% and increases or stays the same thereafter. Note that it never decreases. The cumulative distribution function gives probability of occurrence of medication error on a date and prior to it. For example, 7% of medication errors occur on the second day or prior to it. The cumulative distribution function changes in steps that are equal to the probability of the event at the last time period. For example, the increase in the cumulative distribution function by going from day 1 to day 2 is 6%, which is equal to the probability density function associated with day 2.

EXPECTED VALUE

When faced with uncertain events, an expected value can be calculated. This value is what we anticipate to occur on repeated observations of the event. The expected value can be calculated from the probability density function. It is calculated by multiplying the probability of the event by its value and summing across all possible values of the event.

Table 2

CALCULATION OF EXPECTED DAY FOR MEDICATION ERROR

Day	Probability density function	Day times Probability
1	0.01	0.01
2	0.06	0.12
3	0.04	0.12
4 184[a]	0.89	164
Total		164

[a]Halfway between 4th and 365th day.

$$E(x) = \sum_{i=1,\ldots,n} P(x = i)i$$

where x is a random variable marking a day in our data, i marks a particular day, $E(x)$ is the expected value for the event x, and $P(x = i)$ is the probability of event x occurring on day i.

This formula shows that the expected days to an event is the sum of the product of the day times the probability of event occurring on that day.

For the data in Table 1, to calculate the expected value, we first multiply the value of each day by the probability of the event occurring on that day (this gives the third column in Table 2). In the first row of Table 2, probability of medication errors is multiplied by 1 chance to obtain 0.01. In the second row, 6% is multiplied by 2 to obtain 0.12, and so on. The expected day of medication error is the sum of the products in third column. In this example, it is 164th day. Therefore, the most likely day to expect a medication error is on day 164.

Knowing the probability density function is a very important step in deciding what to expect. It is the building block on which the rest of the analysis rests. If there are errors in estimation of the probability density function, there will be errors in conclusions derived from it. It is important to get an accurate measure of the probability density function. Yet, a full specification of the probability for every day may be onerous. A simpler way to estimate a probability density function is to assume a general shape for the probability function and use a handful of data to estimate the parameters of the function. A great deal of thought has gone into recognizing different probability density functions. The most common probability density functions for discrete variables are Bernoulli, Binomial, Geometric, and Poisson functions. We will describe each of these functions and explain the relationships among them. These functions are useful in describing a large number of events, for example, the probability of wrong side surgery, the time to next gun shot in a hospital, the probability of medication errors over a large number of visits, the arrival rate of security incidences, and so on. If our focus remains on events that either happen or do not happen, then these 4 distribution functions are sufficient to describe many aspects of these events.

BERNOULLI DENSITY FUNCTION

Bernoulli density function is often used to construct more complex distribution functions. It assumes that 2 outcomes are possible. Either the event occurs or it does not. In other words, the events of interest are mutually exclusive. It also assumes that the possible outcomes are exhaustive, meaning that at least 1 of the 2 outcomes must occur. In a Bernoulli density function, the event occurs with a constant probability of P. The complement event occurs at probability of $1 - P$.

$$f(\text{event occurs}) = P$$
$$f(\text{event does not occur}) = 1 - P$$

By the word "constant probability P," we mean that this frequency is not likely to change—typically because of some underlying change in the process that produces the event. For example, the probability of medication error on any particular visit may be assumed to be P; and this probability may be assumed to be constant from visit to visit if the underlying care processes have not changed.

Typically, it is assumed that the probability is calculated for a specific number of trials or a specific period of time. For example, Figure 1 shows the density and cumulative distribution function for a facility where patients have a 5% chance of elopement per day.

The bars in Figure 1 show the density function and the cumulative distribution is shown as a straight line.

Many events have Bernoulli distributions. For example, wrong side surgery has a Bernoulli distribution. Either it happens or it does not. Similarly, wrong blood transfusion, medication error, fire in operating

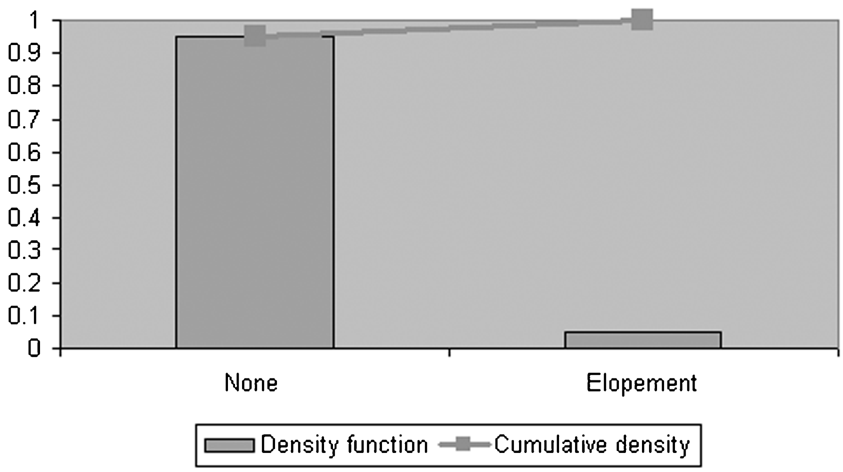

Figure 1. Example of a Bernoulli density and cumulative distribution function.

room, and many other sentinel events have a Bernoulli distribution: either they happen or they do not. Some events that may seem to be mutually exclusive and exhaustive may not be so. At first glance, it may seem that a patient is either alive or dead and, therefore, this variable meets the definition of Bernoulli function but in closer examination, we may run into patients whose brains are dead but they are kept alive. These types of examples contradict the assumption of mutually exclusive and exhaustive events. In checking to see whether Bernoulli assumptions are met, it is often possible to find contradictions. What seems at first black and white may have shades of grey. Sometimes, the assumptions of absence or presence of a sentinel event are accepted even though the event has different degrees of presence.

GEOMETRIC DENSITY FUNCTION

Think of a situation in which a Bernoulli event is repeatedly tried. For example, every day and in every visit, there is a chance that a medication error might occur. This is a repeated occurrence of a Bernoulli trial. For another example, in every surgery, there is a chance that fire may break out. In this sense, any adverse sentinel event that is either present or absent can be thought of as repeated Bernoulli trials.

Assume further that in these trials, the probability of occurrence of the event is not affected by its past occurrence, in other words that each trial is independent of all others. This assumption makes sense if after one sentinel event we do not change the process and reduce the probability of future sentinel events. Independence cannot be assumed for all repeated trials. For example, probability of contagious infection changes if there was an infection in the prior day. Therefore, independent trials cannot be assumed in this situation. But in many situations it can and when we can make this assumption, there is a lot we can tell about the probability function under this assumption.

If a Bernoulli event is repeated until the event occurs, then the number of trials to the occurrence of the event has a Geometric distribution. The Geometric density function is given by multiplying probability of 1 occurrence of the event by probability of $(k - 1)$ non-occurrences that should precede it.

$$f(k) = (1 - P)^{k-1} P$$

To help you think through why that is the case, let us start with looking at an example of repeated independent trials. In this situation, we have 3 repeated trials for tracking patients' elopement over time (Fig 3). We are assuming that the probability of elopement does not change if one patient has eloped in the prior days. On day 1, the patient may elope or not. On day 2, the same event may repeat and another patient may elope. The process continues until day 3. As you can see, the patient may elope on different days and on each day, this probability of elopement on that day is constant and equal to values on prior days.

The Geometric density function gives the probability of kth patient being the first to elope. For that to occur in $k - 1$ occasions, there should have been no elopement. The first part of the function calculates the joint probability of no elopement in $k - 1$ occasions; this is the probability of no elopement, $1 - P$, repeated $(k - 1)$ times. The last part of the equation calculates the probability of elopement in kth occasion, which is simply P.

An interesting property of Geometric probability density function is that the expected number of trials prior to occurrence of the event is given by dividing 1 by the probability of the occurrence of the event in every trial:

Expected number of trials to first occurrence of the event $= 1/\text{P}$

Expected number of trials to and including first occurrence of the event $= 1 + 1/\text{P}$

As shown by Alemi,[1] this fact is used to estimate probability of rare events that might occur only once in a decade. If an event has occurred once in the last decade, then 3650 days have passed before the event has occurred. Then the daily probability of the event is 1/3651, which is 3 in 10 000, a very small probability indeed. Despite the fact that this probability is very small, we are very confident about its accuracy because we have not observed the event more than once in 3650 days of observations.

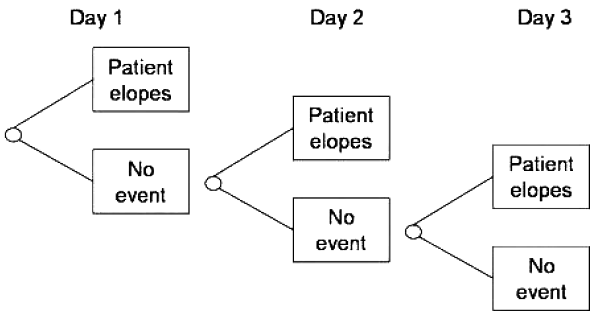

Figure 2. Repeated trials.

BINOMIAL DENSITY FUNCTION

In repeated independent trials of Bernoulli event, the binomial density function gives the probability of having k occurrences of the event in n trials. Take the repeated trials in Figure 2, there are several ways for having 2 patients elope in 3 days: patients elope on days 1 and 2 and not on day 3, patients elope on days 1 and 3 and not on day 2, or patients elope on days 2 and 3 and not on day 1. In all 3 instances, the probability is P^2 $(1 - P)$ but the days on which the patients elope are different. To properly account for number of possible ways of arranging k items in n possible slots, we need to use a combinatorial. This is why, the binomial density function looks like the following:

$$f(k) = \frac{n!}{k!(n-k)!} P^k (1 - P)^{n-k}$$

In this formula, $n!$ is known as n factorial. A factorial of a number is equal to product of all numbers less than or equal to it, that is,

$$\text{Number}! = 1 \times 2 \times 3 \times \cdots \times (\text{Number} - 1) \times \text{Number}$$

The proportion $n!/\{k!(n-k)!\}$ counts the number of different ways in which k occurrence of an event might be arranged in n trials. The term P^k measures the probability of k occurrences of the event and the term $(1 - P)^{n-k}$ measures the probability of $n - k$ non-occurrence of the event.

Figure 3 provides an example density function for binomial distribution based on repeated independent Bernoulli trials with probability of .5. In this figure, you see a binomial distribution with 6 trials. There are 7 possibilities. Either the event never occurs or it occurs 1, 2, 3, 4, 5, or 6 times. The probability density function shows the likely occurrences. The most likely situation for the event to occur is 3 times in 6 days. This is also the expected value and can be obtained by multiplying the number of trials by the probability of occurrence of the Bernoulli event. Note that the distribution is symmetric, although as we will see shortly when $P < .5$, then the distribution gets skewed to the left.

Let us now go back to the patient elopement example, where the dail probability of elopement was 5%. Over a 6-day period, we are most likely to see no patient elopement (Fig 4). There is a 23% chance of seeing 1 elopement and there is a 3% chance of seeing 2 elopements. Note how the distribution is skewed to the left. This is always the case when $P < 50\%$. The more it is smaller than 50%, the more it would be skewed to the left.

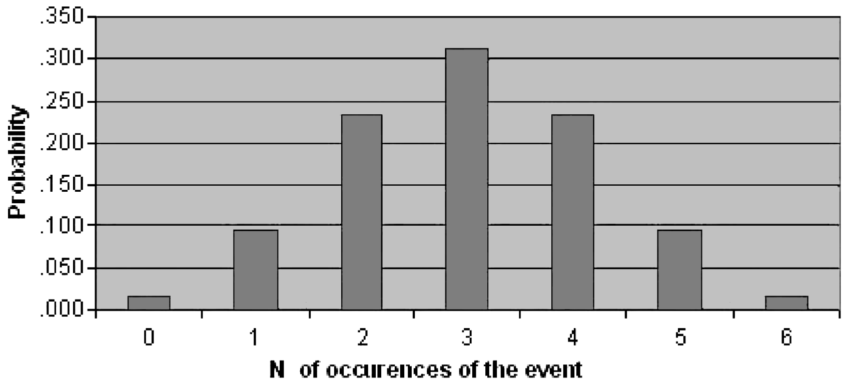

Figure 3. Binomial distribution for repeated independent Bernoulli functions with $P = .5$.

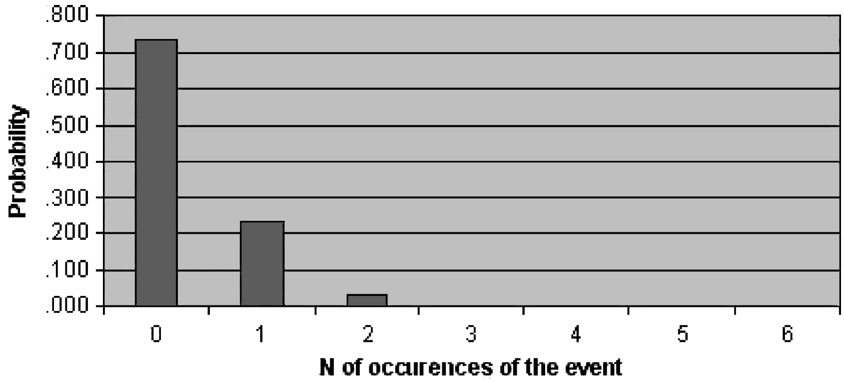

Figure 4. Binomial distribution for repeated independent Bernoulli trials with $P = .05$.

In a binomial distribution, the expected value is calculated from the following formula:

$$E(\text{occurrence of the event}) = nP$$

where n is the number of trials and P is the probability of occurrence of the event in 1 trial.

Now we can answer questions such as how many patients might have an adverse event in a defined period of time. For example, if the daily probability of elopement is .05, the expected number of elopement over a year is $.05 \times 365 = 18.25$ persons. If we want to claim that some interventions have reduced the probability of elopement, then we should see a lot less than 18 persons elope in a year. If we observe that on average, only 2 persons elope in a year, then we can calculate the probability of elopement:

$$P = \frac{2}{365} = .005$$

Note that this probability is smaller than our initial estimate by a factor of 10 times. Therefore, it may be reasonable to assume that we have indeed reduced the rate of elopement. One advantage of using probability models to track sentinel events is that it gives us a precise measure of our success. Tests of significance can be used to see whether the postintervention probability of elopement is statistically different from preintervention estimate.[5]

POISSON DENSITY FUNCTION

Where the number of trials is large and the probability of occurrence is small, Poisson distribution approximates binomial distribution. In risk analysis, this occurs often. Typically, we are looking at a large number of visits or days, and the sentinel event has a very low probability of occurrence. In these circumstances, the number of occurrence of the event can be estimated by Poisson density function:

$$f(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

In this formula, the Greek letter λ is pronounced as lambda; it is the expected number of events in n days, where n is large and P, the daily probability of the event occurring, is small. This expected value can be calculated as follows:

$$\lambda = nP$$

In the Poisson formula, k is the number of occurrences of the sentinel event and e is a constant equal to 2.71828, the base of natural logarithms. Poisson distribution can be used to estimate number of sentinel events over a large period of time. Of particular interest is calculating the probability of sentinel event not occurring during the next time periods; ie, $k = 0$:

$$f(0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-\lambda}$$

Table 3

DEFINITION OF TERMS[a]

Function name	Formula	Definition	Calculated from
Probability distribution function (PDF)	$P(X = t)$	Probability of event occurring at time *t*.	
Cumulative distribution function (CDF)	$P(X \leq t)$	Probability of the event occurring prior to or at time *t*.	Sum of PDF up to time *t*
Survival function (SF)	$P(X > t)$	Probability of the event not occurring prior or at time *t*.	1 − CDF
Hazard function (H)	$P(X = t \mid X \geq t)$	Probability of the event occurring at time *t*, given that it has not occurred prior to this time	PDF/SF

From Cox.[2]

For example, if wrong side surgeries have occurred once in the last 5 years, then as discussed under the Bernoulli density function, the daily probability of wrong side surgery is given as follows:

$$P = \frac{1}{365 \times 5} = 0.0005$$

In a month, this leads to 0.015 expected wrong side surgeries:

$$\lambda = nP = 30 \times 0.0005 = 0.015 \text{ wrong side surgeries}$$

The probability of having no wrong side surgery during the next month is

$$f(0) = e^{-\lambda} = 2.71^{-0.015} = 0.9852$$

Therefore, the probability of having 1 or more wrong side surgeries during the next month is $1 - 0.9852 = 0.0148$.

HAZARD FUNCTION

A hazard function provides the conditional probability of an adverse event occurring if it has not occurred to date. It tells us what might happen, given that nothing has happened to date. It can be calculated from probability density and cumulative distribution functions. Recall the definition of probability density function. This is the probability that the event of interest will occur at time period "*t*." The cumulative distribution function is the probability that the event would occur prior to or at time *t*. It is easy to calculate the cumulative distribution function from the probability distribution functions. The cumulative distribution function is the sum of the probability distribution function for current and prior time periods.

Survival function gives the probability of surviving till after time period "*t*." Note the survival function is the complement of cumulative distribution function and can be calculated as 1 − (cumulative distribution function). Now, we are ready to calculate the Hazard function. It is defined to be the probability of an event occurring at time period "*t*," given that it has not occurred prior to that time period. The hazard function is calculated as the ratio of the probability density function divided by the survival function. Table 3 shows a summary of these definitions.

When the sentinel event is rare, the hazard function is essentially the same as the probability distribution function. Think about it, when something is rare, the survival function is near 1 and the hazard rate and probability distribution function are nearly equal.

EXAMPLE OF FIRE IN SURGICAL UNIT

An example will demonstrate the calculation of hazard rate from probability density functions (see

Table 4

AN EXAMPLE FOR CALCULATING HAZARD ASSOCIATED WITH FIRE IN SURGICAL ROOMS

Symbol calculation	Probability distribution function (PDF) estimated	Cumulative distribution function function (CDF), = Σ PDF	Survival function (SF), (1 − CDF)	Hazard function (HF), PDF/SF
Year 1	0.10	0.10	1.00	0.10
Year 2	0.10	0.20	0.90	0.11
Year 3	0.01	0.21	0.80	0.01
Year 4	0.01	0.22	0.79	0.01
Year 5	0.01	0.23	0.78	0.01
Year 5 or more	0.77	1.00	0.00	1.00

Table 4 for the calculations). Suppose that because of planned changes in oxygen equipment, fires will occur in the surgical units at the rate of 0.10 for the first 2 years and 0.01 thereafter. Note that in this example the probability of failure anytime during the 5 years does add up to 1.0, meaning that at least one fire will occur in the next 5 years. Given this distribution, we want to understand what is the probability that if the

Table 5

PROBABLE CAUSE OF UNAUTHORIZED DISCLOSURE

	Disgruntled employee sells information		Clinician discusses patient information in social gathering		Any unauthorized disclosure	
	Day of incidence	Days to next event	Day of incidence	Days to next event	Day of incidence	Days to next event
	May 11, 2004	1290	Dec 5, 2006	387	May 11, 2004	938
	Nov 22, 2007		Dec 27, 2006		Dec 5, 2006	352
					Nov 22, 2007	35
					Dec 27, 2007	
Average time to event		1290		387		442
Daily probability of unauthorized disclosure		0.0008		0.0026		0.0023
Expected number of monthly disclosures		0.0236		0.0786		0.0689
Probability of no unauthorized disclosure next month		0.9767		0.9246		0.9336
Hazard rate (probability of unauthorized disclosure next month)		0.0233		0.0754		0.0664
Attributable risk		23.60%		76.40%		100%

fire has not broken out for 2 years and it will occur in the 3rd year or more broadly, what is the hazard function associated with fire in year 2.

First we need to calculate the cumulative distribution function. We need to calculate a sum of the probability for the current and the prior years. Note that this is the sum of the cumulative probability in the immediate prior year and the probability distribution in the current year. For example, the numbers in Table 2 suggest that the probability of the fire either in year 1 or in year 2 is 20%.

The survival function is 1 − (prior year's cumulative distribution). Probability of surviving at the start of year 1 is 1. The probability of surviving until the start of year 2, in this example, is .90, as 10% of surgeries will have fire in the prior year. The probability of surviving till the start of year 3 is 80%, as 20% will have fire in the first 2 years while we are waiting for better equipment. Note that the survival probability or the chances that there will be no fire decreases with time.

Now we can calculate the Hazard function. This is the function that calculates the probability of the event occurring this year if it has not occurred in the prior years. This is calculated as the ratio of the probability function and the survival function. Note that the hazard function drops in year 3, the year we expect to have better equipment. If we have survived up to this point without a fire, we now face the lower probability of fire associated with better equipment. So, now we can answer the question asked earlier regarding what the probability of the fire in year 3 is, if there has not been one until then. This probability is 1%.

DIFFERENT SOURCES OF HAZARDS

If we assume that various causes are independent of each other, then the hazard function from all sources can be calculated as the sum of hazard function from each source. This is a very helpful concept because it allows us to calculate the total impact of multiple causes. If H shows the combined hazard function and h_i shows the hazard associated with cause "i," then the hazard function of the combination can be calculated as follows:

$$H = h_1 + h_2 + \cdots + h_n$$

For example, under assumption of independence, the hazard function for medication error could be the sum of hazards due to fatigued nurse and hazard associated with physician's poor communication skills.

$$H = h_{\text{fatigued nurse}} + h_{\text{physician's poor communication skills}}$$

The combined hazard risk can also be used to calculate the attributable risk to a specific cause. The attributable risk because of a cause is calculated as the ratio of its source specific hazard rate and the hazard function from all sources.

$$AR_i = \frac{h_i}{\sum_i h_i} \times 100$$

An example can help demonstrate the concept of attributable risk. If the hazard function for medication error caused by a fatigued nurse is 1 in 1000 and the hazard function for medication error caused by illegible prescription order is 2 in 1000, what is the attributable risk to the nurse and to the physician? Using the formula for attributable risk, we calculate this as the ratio of nurse's hazard rate and the total hazard rate. In this case, it is 33%. In contrast, 66% of the medication error risk is attributable to the physician's poor prescription writing skills. The most probable cause of the event is the physician's ineligible writing. In this fashion, relative attribution can be made to separate risks.

Correct attribution, of course, is important to priorities one sets for remedies to the problem. If 66% of risk is due to ineligible prescriptions, then it might be best to resolve this problem first before addressing the problem of fatigued nurses. Of course, the best action is to address both causes but if we cannot and need to set a priority, the analysis shows how to focus on key causes that are more responsible for the observed sentinel events.

FINDING THE MOST PROBABLE CAUSE

Suppose that review of incidence reports in the past 3 years has identified 2 types of unauthorized disclosure of patient information: disgruntled employee selling information (done May 11, 2004, and November 22, 2007), and clinician discussing patient information in social settings (done December 5, 2006, and December 27, 2007). If the monthly incidence report indicates a new unauthorized disclosure, what is the most likely cause? The first step is to calculate the time to each type of unauthorized disclosures: 1290 days for disgruntled employees selling information and 387 days for clinicians discussing patient information in social gatherings. This provides an estimated daily probability of the event. Note that this estimate is different than dividing the number of events by 3 years. Such a division would have resulted in the same rates for the 2 causes while one has a 3 times more time between its reoccurrences. The way we have estimated the daily rate reflects the time between events and not just the number of events. This leads to the probability of disclosure due to socialization to be 3 times higher than its probability due to employees selling information.

Once the daily rate has been established, we estimate the daily number of unauthorized disclosures and use this in calculating the probability of no-unauthorized disclosure in next month. The monthly hazard rate is estimated as 1 − (probability of no-unauthorized disclosure). The relative contribution of each hazard rate is calculated as the hazard rate divided by the sum of hazard rates from any source. Here are the calculations for attributable risk (AR), associated with a disgruntled employee selling information:

$$\lambda_{\text{Selling}} = nP = 30.5 \times 0.0008 = 0.0236$$

$$f(0) = e^{-\lambda} = 2.71^{-0.0236} = 0.9767$$

$$h_{\text{Selling}} = 1 - 0.9767 = 0.0233$$

$$AR_{\text{Selling}} = \frac{h_{\text{Selling}}}{h_{\text{Selling}} + h_{\text{Socialization}}} \times 100 = \frac{0.0233}{0.0233 + 0.0754} = 23.60\%$$

The socialization of clinicians is much more likely to lead to unauthorized disclosure than disgruntled employee selling the information. This type of information can be used to set priority of where risk reduction efforts should focus on.

CONCLUSIONS

Hazard functions are key to risk analysis. These functions can be used to calculate how much of risk can be attributed to different causes. These functions can be used to predict probability of adverse events in next time period. This tutorial has shown how hazard functions can be calculated from discrete probability density functions.

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